Applied Mathematics and Computation 218 (2012) 8242–8259

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# A sequential niching memetic algorithm for continuous multimodal function optimization

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ARTICLE INFO

*Keywords:*
Memetic algorithms
Genetic algorithms
Multimodal function optimization
Multiple optimal determination
Niching
Local learning

ABSTRACT

This work discusses a sequential niching algorithm for multiple optimal determination. The procedure consists of a sequence of genetic algorithms (GA) runs, which incorporate a gradient-based hill-climbing algorithm, and make use of a derating function and of niching and clearing techniques to promote the occupation of different niches in the function to be optimized. Thus, the algorithm searches the solution space eliminating from the fitness landscape previously located peaks forcing the individuals to converge into unoccupied niches. An effective search of the solution space is stimulated incorporating in the algorithm stages dedicated to find new promising domains in the variable space and stages that exploits the located promising regions. Unlike other algorithms the efficiency of the sequential niching memetic algorithm (SNMA) proposed in this work is not highly sensitive to the niche radius. Performance measurements with 37 standard test functions of dimensions ranging from 1 to 100 show that the SNMA has very good scalability and outperforms other algorithms in accurately locating multiple optima, both global and local.



## 1. Introduction

Classical techniques of nonlinear optimization, whether first or second order methods, such as steepest descent, conjugate gradients, Newton and quasi-Newton techniques guarantee at most, the convergence to the bottom (top) of whatever valley (hill) they started in. In complex nonlinear problems where many local optima may exist, finding the global optimal solution is difficult with these methods since many different starting points in the solution space must be tried before finding a global or satisfactory local optima [1]. Guided stochastic search algorithms such as simulated annealing [2] (SA) and evolutionary computing [3,4], possess good global search capabilities but show poor convergence characteristics since they make use only of function evaluations and thus it is not possible to tell how close a candidate solution is to the optimal. Hybridization of genetic algorithms (GA), a form of evolutionary computing, with SA have been proposed by several authors [5,6], nevertheless as with other stochastic algorithms, they still lack good convergence properties.

On the other hand, it is widely recognized by the evolutionary computing (EC) community that the combination of GA with local search, known as memetic algorithms [7] greatly improves the ability of evolutionary algorithms to accurately locate optimal solutions in function optimization problems. Many problems of interest in scientific and engineering fields require the determination of all optimal solutions, both global and local, of a multimodal function in a given region of the solution space. Examples of this is the determination of the minima associated to the energy surfaces of interacting boson models, i.e. the expectation values of algebraic Hamiltonians with respect to their corresponding coherent states, which are used as variational trial states [8]. In general these models consider a nucleus formed by two sets of bosons one with angular

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doi:10.1016/j.amc.2011.05.051

momentum zero (s-bosons) and the other with angular momentum two (d-bosons). The accurate location of the minima are very important to give a geometric interpretation of these models. These minima yield information regarding the energies of the ground and excited states as well as on nuclear shape-phase transitions [9–11]. Up to our knowledge some authors have approached this problem by dividing the phase space coordinates, defined by the coherent state, in fine meshes and identifying the local minima. The coordinates of these minima are then refined using conjugate gradient searches [12]. This can be a time-consuming task when the meshes are small and the dimensions of the search space is large. The SNMA proposed in this work, which is an extension of a previous work [13], can be used to search for all the local minima of the energy surfaces, in a more systematic and efficient way.

Memetic algorithms aim at locating a promising area in the solution space and use local optimization techniques to intensify the search within that region. There are a large number of works in this respect, some of the most important are discussed very briefly below.

Chelouah and Siarry [14] use a Nelder–Meade algorithm [15] for local learning in GA, which requires only function evaluations. Gudla and Ganguli [16] use a conjugate gradient technique to improve on the best candidate found after independent series of GA searches. Wei and Zhao [17] use a niche technique, known as *clearing*, proposed by Petrowski [18] to eliminate similar solutions in search space and merged it with a Nelder–Meade method to improve the performance of GA for global optimization. Hacker et al. [19] proposed a hybridization, based on local topography of the solution space, that switches between GA and local search (conjugate gradients) for more accurate global optimal determination. Espinoza et al. [20] proposed an algorithm that switches from genetic to gradient descent every certain number of generations into the GA and randomly performs local search with individuals in the population. Other algorithms deal with GA mechanisms. For example Lozano et al. [21] proposed a real-coded memetic algorithm based on crossover hill-climbing that maintains a pair of parents and perform crossover on this pair until a prespecified number of offspring is reached. The best offspring is then selected to replace the worst parent if it is better. The process repeats a certain number of iterations yielding two final parents. Recently Shelokar et al. [22] proposed an improved particle swarm optimizer for global optimization of multimodal functions hybridized with an ant colony method to update position of particles.

The techniques developed to approach the problem of locating multiple optimal of a multimodal function can be classified as: parallel subpopulation methods and iterative techniques. Parallel subpopulation methods divide the total population into subpopulations (species) that evolve in parallel, searching different regions of the solution space. Iterative techniques on the other hand aim at locating multiple optima by repeating the same algorithm several times avoiding convergence to the same solutions by prohibiting the algorithm to search those regions of the solution space that have already been explored.

In most of the parallel subpopulations methods the species evolve around *seeds* or *dominating* individuals, which are the fittest members of the population within a prespecified radius called *species distance*. The species distance specifies the maximum distance between two individuals for which they are considered to be similar. A technique called *species conserving GA* (SCGA) have been proposed by Li et al. [23] which conserves the dominating individuals of each species by copying them into the next generation. The algorithm applies GA mechanisms to all members of the generation and compare the new members with the species seeds; the seeds are then either conserved or superseded by better members of the current generation. Another algorithm called *coevolutionary sharing*, inspired in a model of economics was proposed by Goldberg and Wang [24]. This method introduces business and costumers populations; the population of customers is divided in sets by finding the closest business to each costumer. Both populations evolve in parallel, the customers using a fitness sharing evaluation function and the business a fitness function equal to the sum of all the raw fitness values of its customers. In addition it was introduced a strategy to replace a business, in the business population, using its customers set. The role of the business population is to locate the niches of the multimodal function. Since a minimum number of individuals are necessary to accurately explore and locate each optimum. If many optimal must be located or the dimensionality of the problem is high, a large population is required, which may be a drawback.

Niche techniques have been used to promote population diversity. There are two basic approaches to niching: *crowding* and *fitness sharing*. In *crowding* an individual is compared with members of a randomly chosen subpopulation and the most similar member of the population is replaced [25]. *Fitness sharing* on the other hand, defines a degradation of the fitness of an individual due to the presence of neighboring individuals within a distance known as niche radius; the underlying idea is to give all local and global optimal solutions an equal opportunity to survive.

An important iterative method is the sequential niching GA (SNGA) proposed by Beasley et al. [26] This technique consists of a sequence of GA searches each aiming at locating a different optimum solution. Once the first optimal was found by means of a GA, the second sequence searches the solution space eliminating that peak from the fitness landscape by penalizing individuals that may stray into that region. The individuals are then forced to converge into an unoccupied niche, which in the following sequences is also assumed to be filled. The process should contain at least as many sequences as optima are expected in the function. The difference between fitness sharing and this technique is that fitness sharing dynamically modify the fitness landscape according to the location of all individuals. On the other hand, the SNGA modify the fitness landscape according to the location of all the solutions previously found, discouraging individuals from re-exploring those areas. This is achieved by multiplying the raw fitness function by a derating function whenever an individual is closer than a niche radius to any of the previously found solutions. However, the use of the derating function introduces additional peaks around the corresponding optimum. To assign a niche radius Deb and Goldberg [27] assume the global optima are evenly distributed throughout the search space. This is an approximation since in most problems the optimal solutions are not uniformly distributed. When a small niche is used spurious peaks around the optimal are introduced, which must be discarded