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The EOQ model – A dynamical system

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ABSTRACT

The paper deals with an inventory model to determine the retailer's optimal order quantity for similar products. It is assumed that the amount of display space is limited and the demand of the products depends on the display stock level and the initiatives of sales staff where more stock of one product makes a negative impression of the another product. Also, the replenishment rates depend on the level of stocks of the items. The objective of the model is to maximize the profit function by trading off inventory costs, purchasing costs, cost of the effort of sales staff considering the effect of inflation and time value of money by Pontryagin's Maximal Principles. The stability analysis of the concerned dynamical system has been analyzed. The sensitivity analysis of a suitable example is also carried out.

1. Introduction

The impact of visual attraction reflects on market demand. A large display in a shopping mall naturally encourage the people to purchase more. According to Levin et al. [1], "large piles of consumer goods displayed in a supermarket will lead the customer to buy more". Silver and Peterson [2] also noted that sales at the retail level tend to be proportional to the amount of inventory displayed. Baker and Urban [3] considered a power-form inventory-level-dependent demand rate that would decline along with the stock level throughout the entire cycle. Ghosh and Chaudhuri [4] developed an order level inventory model for a deteriorating item with two levels of storage stock-dependent demand pattern. Datta and Pal [5] modified the model of Baker and Urban [6] by considering the demand rate that depleted to a given level of inventory, beyond which it was a constant. By their assumption, not all customers are motivated to purchase goods by huge stock. Some of the customers might arrive to purchase goods because of its goodwill, good quality or facilities in spite of low stock level. Zhou et al. [7] considered a decentralized two-echelon supply chain where the demand of the product was dependent on the displayed inventory. Soni and Shah [8] formulated an optimal ordering policies for retailer in trade credit environment where demand was partially constant and partially dependent on the stock. Goyal and Chang [9] formulated an ordering-transfer inventory model while the storage capacity was limited and the demand rate dependent on stock displayed. They determined the retailer's optimal ordering quantity and the number of transfer per order from the warehouse to the display area for maximizing the average profit per unit yielded by the retailer. Hsieh et al. [10] extended the model of Datta and Pal's [5] model, allowing partial backlogging. Hsieh and Dye [11] provided some useful properties for finding the optimal replenishment schedule with stock-dependent demand under exponential partial backlogging. Min et al. [12] investigated an EOQ model for perishable items under permissible delay in payments where demand of the products varied linearly with the level of stock. Sajadieh et al. [13] developed an integrated vendor–buyer model for a two-stage supply chain, assuming demand of the products as a positive power function of displayed inventory. Jolai et al. [14] developed a framework to derive optimal production over a fixed planning horizon for perishable items that follows a two-parameter Weibull distribution with a stock-dependent demand rate under inflationary conditions. Gupta et al. [15] presented an inventory model by considering

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the impact of marketing strategies such as pricing and advertising on three component demand rate for a single item with imprecise inventory costs. In their model, the demand of the commodities is dependent on selling price, frequency of advertisement and displayed stock level in a show room/shop. The research articles related to the stock-dependent demand pattern are Urban [16], Pal et al. [17], Goh [18], Padmanabhan and Vrat [19], Sarker et al. [20], Datta and Paul [21], Balkhi and Benkherouf [22], Chang [23], Hou and Lin [24], Min and Zhou [25], among others.

Now-a-days, various types of promotional efforts such as price discount, buy two get one free, quantity discount, and warranty period provoke the customers to buy more. Goyal and Gunasekaran [26] presented an integrated production–inventory–marketing model for determining the economic production quantity (EPQ) and economic order quantity (EOQ) for raw materials in a multi-stage production system. They considered the effect of different marketing policies such as the price per unit product and the advertisement frequency on the demand of a perishable item. Many studies focused on the effect of promotions on sales using store or market level data [27, 28, 29]. Sun [30] made relation of the customers behavior with different types of promotions and identified that promotions had a strong impact on stronger brands. Based on the work of Divakar et al. [29], Ramanathan and Muyldermans [31] applied structural equation modeling to investigate the impact of promotions and other factors on the sales of soft drinks. Sana and Chaudhuri [32] developed an inventory model for stock with advertising sensitive demand. Sana [33] investigated an interesting multi-item EOQ model for deteriorated and ameliorating items when the time varying demand is influenced by enterprizes' initiatives like advertising media and the effort of sales staff. Recently, Sana [34,35] developed a dynamical system of similar products for different types of demand functions. In these models, the joint effort by sales staff is considered and the inventory cost per unit product is fixed.

It is a common practice that the inventory holding cost per unit product per unit time is assumed to be constant. Baker and Urban [6], and Datta and Pal [5] developed inventory models by considering constant holding cost. Muhlemann and Valtis Spanopoulos [36] first introduced the concept of variable holding cost. Stock holding cost may be linear or nonlinear function of stock and its holding period. Weiss [37] assumed the holding cost as a nonlinear function of the time-span for holding the inventory stock. Goh [18] focused on both time-dependent and stock-dependent holding cost in his model. An inventory system with a variable holding cost is studied by Alfares [38] where demand of an item depends on the on-hand inventory. Urban [39] framed an inventory model while the stock holding cost is taken as a discrete variable whereas a time-varying holding cost is presented by Roy [40].

In the last decade, the inflation and time value of money ruin the global economy. Consequently, it's effect on economical analysis in any businesses should be considered. Buzacott [41] was the first in this direction who derived expressions for the optimal order quantity, considering inflation and time value of money. In this direction, the great works of Biermann and Thomas [42], Datta and Pal [43], Bose et al. [44], and Hou [45] are worth mentioning, among others.

In this paper, a dynamical system of differential equation of two types of similar products has been considered when the demand rate of each product depends on the initiatives of sales staff and level of stock of the products. The replenishment rates of the products are also dependent on the on-hand inventory. The huge stock of one product decreases the demand of the another product.The inventory costs per unit products are nonlinear functions of the respective on-hand stocks. The stability analysis of the system has been done. Finally, a profit function considering inventory costs, purchasing costs, selling prices, cost of effort has been optimized by Pontryagin's Maximal Principles.

The rest of the paper is organized as follows: Section 2 provides the notation of the parameters and variables of the model. Formulation of the model is described in Section 3. Boundedness of the dynamical system is proved in Section 3.1. Local stability and global stability analysis have been done in Sections 3.2 and 3.3 respectively. Optimal goal of the policy is obtained in Section 3.4. To test the proposed model, numerical example is given in Section 4 and its sensitivity analysis is discussed in Section 4.1. Section 5 provides conclusion and future extension of the proposed model.

2. Notation

The following notation are considered to develop the model:

Notation:

$X(t)$ – on hand stock of item 1 at time t.
$Y(t)$ – on hand stock of item 2 at time t.
D_x – demand rate of item 1.
D_y – demand rate of item 2.
R_x – replenishment rate of item 1.
R_y – replenishment rate of item 2.
L_x – maximum allowable amount of displayed capacity of item 1.
L_y – maximum allowable amount of displayed capacity of item 2.
L – maximum allowable amount of displayed capacity ($L = L_x + L_y$) of items 1 and 2.
$[E_1(t), E_2(t)]$ – effort functions of the advertising and the initiatives of sales staff for items 1 and 2 at time t, respectively.
p_x – purchasing cost (in $ currency) per unit item 1.
p_y – purchasing cost (in $ currency) per unit item 2.
h_x – inventory holding cost (in $ currency) per unit item 1 per unit time.