Applied Mathematics and Computation 218 (2012) 8925–8936

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Role of instant nutrient replenishment on plankton dynamics with diffusion in a closed system: A pattern formation

Joydip Dhar [a], Randhir Singh Baghel [b,*], Anuj Kumar Sharma [c]

[a] ABV-Indian Institute of Information Technology & Management, Gwalior, M.P., India
[b] School of Mathematics and Allied Science, Jiwaji University, Gwalior, M.P., India
[c] Department of Mathematics, L.R.D.A.V. College, Jagraon, Punjab, India

ARTICLE INFO

Keywords:
Aquatic ecosystem
Reaction–diffusion mechanism
Diffusion driven instability
Spatiotemporal chaos
Spatial pattern formation
Higher order stability analysis

ABSTRACT

A mathematical model is proposed to study the role of instantaneous nutrient recycling on the plankton ecosystem. In this model, we include three state variables namely, nutrient biomass, phytoplankton and zooplankton population with Holling type II response function for the population density transformation from phytoplankton to zooplankton. It is obtained that the local stability of different equilibrium depends on the nutrient supply rate to the phytoplankton for the temporal system and also existence of the oscillatory behavior of the temporal system is established by using Bendixson–Dulac criteria. In the spatiotemporal model, we also determine the diffusion-driven instability condition, with the numerical support for the effect of diffusivity coefficients on chaotic behavior of the system. Furthermore, we obtained the instability condition for linear and no-linear system from the higher order stability analysis. Finally, we analyze the time evaluation pattern formation of the spatial system. This shows that it is useful to use the reaction–diffusion system to reveal the spatial dynamics in the real world.

1. Introduction

Pollution of fresh water in marine system by anthropogenic sources has become a concern over the last decades [1–3]. Researchers have found out that each tea spoon of ocean water contains 10 million to 100 million of viruses. Viral infection is known to cause a cell lysis in phytoplankton [4]. The measurable level of toxin due to harmful species is responsible for the bloom dynamics. In coastal area, viral disease can infect bacteria and phytoplankton [5]. Virus like particles are found in natural phytoplankton community [4,6]. These virus like particles have also been found in many eukaryotic algae [7]. The parasite modifying behavior has also been exhibited by the infected individuals of host population. This may happen by reducing stamina, disorientation and altering responses in infected population [8]. Killifish (*Fundulus parvipinnis*) tends to come closer to the surface of the sea after being infected, which make them more vulnerable to predation by birds [9]. Viruses have been held responsible for the collapse of *Emiliania* huxleyi bloom in mesocosms [10,6]. Since, viruses have major role in shaping the dynamics of plankton, so many researchers have developed and analyzed different mathematical models [11–13].

The dynamics of rapid (or massive) increase or decrease of plankton populations is an important subject in marine plankton ecology [14]. Generally high nutrient levels and favorable conditions play a key role in rapid or massive growth of algae and low nutrient concentration as well as unfavorable conditions limits their growth. The water must contain high levels of

* Corresponding author.
E-mail addresses: jdhar@iiitm.ac.in (J. Dhar), randhirsng@gmail.com (R.S. Baghel), anujsumati@gmail.com (A.K. Sharma).

0096-3003/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2012.02.052

inorganic nutrients (nitrogen and phosphorus) for the algae to feed on and also water temperature and salinity levels must be within a certain range to be conducive to planktonic growth [15]. A frequent outcome of planktonic bloom formation is massive cell lysis and rapid disintegration of large planktonic populations. This is closely followed by an equally rapid increase in bacterial numbers, and in turn by a fast deoxygenation of water, which could be detrimental to aquatic plants and animals. These blooms also reduce the chance of growth for aquatic vegetation. Hence, studies regarding the pattern of blooms are necessary towards this serious ecological problem [16–18]. Plankton pattern formation is dependent on the interplay of various physical (light, temperature, hydrodynamics) and biological (nutrient supply, predation) factors [19–21]. In nature, it has been observed that the direction of motion of plankton patches does not always coincide with that of the water [22], and as the spatial scale increases above approximately 100 m, phytoplankton behaves successively less like a simple passive quantity distributed by turbulence [23,24]. Similarly, the spatial variability of zooplankton abundance differs essentially from the environmental variability on scales of less than a few dozen kilometers [25]. This indicates that biological factors play an essential role in the emergence of plankton patchiness.

Biological motivation for chemostat-type interactions models have been studied by [26–28] among others. Mathematical analysis of chemostat-type competition and plankton ecosystem have been carried out in [29,30] and global dynamics of a chemostat model with differential death rates was studied by Wolkowicz and Lu [31].

Keeping in view the above discussion, we have studied the role of virus in phytoplankton, which render phytoplankton more vulnerable to predation by zooplankton, by developing two chemostat models of phytoplankton and zooplankton species (see Fig. 1).

In Section 2, we have developed mathematical model and analyzed dynamical behavior of chemostat model of phytoplankton and zooplankton species. In Section 3, proposed the model with diffusion with drive the instability condition analytic as well as numerically and studied the one and two-dimensional space systems. In Section 4, we have studied the higher order stability analysis and finally in Section 5, we summarize our results and discuss the relative importance of different mechanisms for the onset of spatiotemporal chaos and pattern formation.

2. Mathematical model

Motivated from the work of Ruan [30], on zooplankton–phytoplankton–nutrient models with instantaneous nutrient recycling, we purpose a mathematical model, taking $N(t),\ P(t)$ and $Z(t)$ are nutrient biomass, phytoplankton and zooplankton population densities at any time t, respectively. Again, d_1 and d_2, are the per capita death rate of phytoplankton and zooplankton, respectively. In a natural plankton system, water flowing into the system brings in nutrient and outgoing water carries away nutrient from the system. Further, it is assumed that water is carrying away nutrient, phytoplankton along with flow with the same rate. Here, N_0 is the constant nutrient input concentration at any time and D is the water influx rate or washout rate along with nutrient, phytoplankton and zooplankton. The nutrient uptake and grazing rate of phytoplankton by zooplankton is assumed to follow law of mass action. Let a_1 be the initial nutrient uptake rate by phytoplankton and c_1 is the grazing rate of phytoplankton by zooplankton. The conversional rate of the nutrient into phytoplankton is given by a_2 and c_2 is the conversional rate of phytoplankton into zooplankton. Again, the positive feedback term's $(1-a_2)a_1NP,\ \frac{(1-c_2)c_1PZ}{\alpha+P},\ d_1P$ and d_2Z will be recycled into nutrients, i.e., all the losses are being replenished into nutrients. Model equations for the above system are given by

$$\frac{dN}{dt} = D(N_0 - N) - a_1NP + (1-a_2)a_1NP + q\frac{c_1PZ}{\alpha+P} + d_1P + d_2Z, \tag{1}$$

$$\frac{dP}{dt} = a_2a_1NP - \frac{c_1PZ}{\alpha+P} - d_1P - DP, \tag{2}$$

$$\frac{dZ}{dt} = \frac{c_1c_2PZ}{\alpha+P} - d_2Z - DZ, \tag{3}$$

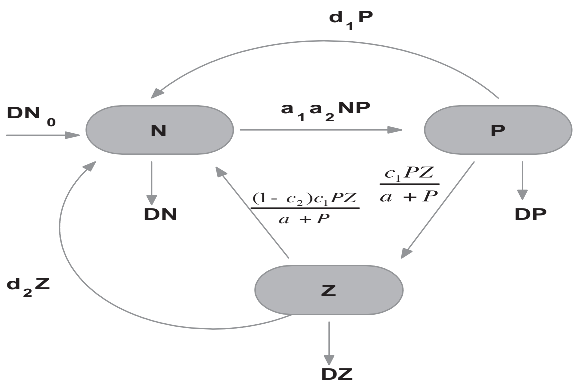

Fig. 1. Schematic diagram.