Applied Mathematics and Computation 218 (2012) 9007–9017

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

A space charge motion simulation with FDTD method and application in negative corona electrostatic field analysis

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ARTICLE INFO

Keywords:
Space charge motion simulation
Finite difference time domain
Corona

ABSTRACT

In this paper, a finite difference time domain based simulation method is presented for the spatio-temporal analysis of space charge motion and the proposed method is applied to negative corona electrostatic field analysis. Drifting and diffusion motion equations of space charges are numerically solved and used in the simulation of corona discharges considering effects of impact ionization, electron attachment, ion–ion recombination and ion–electron recombination. The results obtained from the simulation are discussed.

1. Introduction

Space charges are mobile charge carrier particles such as electrons and ions and hence they are responsible for electrical conduction in electro-physical systems. The numerical simulation of space charge motion in the system contributes to investigation of a wide variety of scientific and engineering problems from nano to cosmological scales, such as conduction in liquids, gases, solids and plasma, electro-biological conduction (neural conduction), and even the ionic activities seen on the sun's corona.

In corona discharge, motion of space charges plays a substantial roll in discharge mechanism. Corona discharge has been found in numerous applications of today's technology [1–11]. The increasing trend in the use of corona discharge applications has renewed interest in basic aspects of the corona electrostatic field. It is therefore becoming more important to understand the mechanisms in corona discharge and the electrical characteristics of complex electrode systems to move forward electron–ion technologies.

Many numerical methods for the analysis of the corona discharge have been proposed in the literature, covering several aspects: Poisson's equation was solved for a finite solution points on the radial axis of coaxial electrodes system by considering impact ionization, photoionization, attachment and detachment of electrons, and ion–ion recombination [12]. In many works, the Finite Element Method (FEM) [13,14] and Finite Difference Method (FDM) [15–17] were applied to iteratively solve current continuity equations over a two-dimensional mesh describing the electrode systems. These methods mainly calculate the charge distribution that satisfies Poisson equation and the continuity equation for a simulated electrode system and they do not directly address simulating the motion of the space charge in an electrode gap. In order to obtain better results, more efficient algorithms have been proposed, such as the method of characteristics (MOC) dealing with the artificial dispersion of space charges [18–20] and hybrid algorithms [21,22] that employ a combination of the Boundary Element Method [21], FEM and FDM. Concerning to space charge motion, the Monte Carlo method has also been applied to simulate the motion of electrons and ions [23,24]. Thus, the temporal and spatial development of space charges and variations in the electrostatic field could be analyzed for the case of bipolar charges. In a recent work, a new 2-D numerical model

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0096-3003/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2012.02.063

incorporating three ionic species for the negative corona discharge was successfully proposed to simulate a series of Trichel pulses by using FEM method for solving Poisson equation and a combined Flux Corrected Transport–FEM method for solving charge transport equations [25].

Previously, the FDTD based numerical analysis methods have been successfully applied to obtain the transient solutions of electromagnetic wave propagation [26,27] and acoustic wave propagation [28] problems. In this paper, we primarily aim to develop a FDTD based simulation methodology for the transient analysis of the space charge motion in a two-dimensional plane representing a medium in inhomogeneous electrical characters. For this propose the spatio-temporal solutions of the motion equations, considering drifting and diffusion of space charges, is found for a discrete domain of space and time. This numerical solution was adopted to the simulation of negative corona electrostatic field. In a negative corona electrostatic field, the effective space charge species are electrons, with negative ions as the negative charge carrier particles, and positive ions as the positive charge carrier particles. In the FDTD simulation developed for inspection of corona discharge, Townsend electron avalanches were initiated by residing seed electrons in the vicinity of a corona electrode. The development of electron avalanches and the generation of positive and negative ions as a result of the impact ionization and electron attachment were observed in the electrode gap of the sphere-plate electrode system. The migration of ion sheets to the electrode was viewed in the simulation.

The proposed method mainly enables us to numerically analyze the spatio-temporal development of multi-charge carrier populations. This specifically demonstrated in numerical analysis of corona electrostatic fields on a spatial plane.

2. Method

2.1. Theoretical background

Space charges motion is modeled in two aspects: one is the drifting movement of charges under an electrical field and the other is diffusion movement of charge carrier particles in an inhomogeneous charge distribution of the medium.

In an electrical field, $\vec{E}$, space charges drift with a drifting velocity [29–31]:

$$\vec{V}_d = \mu \cdot \vec{E}, \tag{1}$$

where, μ is the charge mobility and $\vec{V}_d$ is the drifting velocity, and it can be written with respect to the displacement vector of space charge as $\vec{V}_d = \frac{\partial \vec{d}}{\partial t}$. In this case, space charge drifting motion can be modeled as:

$$\frac{\partial \vec{d}}{\partial t} = \mu \cdot \vec{E}. \tag{2}$$

In an inhomogeneous charge distribution, the diffusion motion of charge carrier particles occurs in accordance with Fick's law as [32]:

$$\vec{J}_d = D \cdot \nabla N, \tag{3}$$

where, $\vec{J}_d$ is the particle current density, the parameters N and D are the particle concentration in space and the diffusion coefficient, respectively. In the diffusion, particle flow conforms to the current continuity equation, which is given as $\frac{\partial N}{\partial t} - \vec{\nabla} \cdot \vec{J}_d = 0$ [12–17,29–31], in this case, the diffusion motion of particles can be expressed as:

$$\frac{\partial N}{\partial t} = D \cdot \nabla^2 N. \tag{4}$$

Eqs. (2) and (4) are numerically solved for the corona field simulation under the following basic assumptions:

(1) Unipolar space charges take effect in the system. Negative charge carriers are electrons and negative ions and positive charge carriers are positive ions.
(2) Drifting and diffusion mobilize the space charges in the medium. Charges are subject to impact ionization, electron attachment, ion–ion recombination and ion–electron recombination during their motion.
(3) Permittivity (ε) and charge mobilities (μ) are constant in the medium.

The impact ionization of electrons with gas molecules results in positive ion and free electron production and it is the main physical process that feeds electron avalanches, initiated with seed electrons. Growing of avalanche electron concentration in the spatial domain was modeled as the following:

$$\frac{\partial N}{\partial r} = \alpha \cdot N, \tag{5}$$

where, α is the impact ionization coefficient [29–31].

Electron attachments with gas molecules result in negative ion production and free electron removal in the medium. It is therefore an effective physical process quenching electron avalanches. In the case of electron attachment, avalanche electron concentration was modeled as the following: