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Codimension-2 bifurcations of coupled BVP oscillators with hard characteristics

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ABSTRACT

This manuscript is to study codimension-2 bifurcations for coupled nonlinear BVP oscillators with hard characteristics. Sufficient conditions are given for the model to demonstrate Hopf, Bogdanov–Takens (BT), double Hopf, and fold-Hopf bifurcations. By computing normal forms of those bifurcations, bifurcation diagrams such as Hopf, homoclinic, double limit cycle, and torus bifurcations are obtained. Some examples are given to confirm the theoretical results.

1. Introduction

In this paper, we study the dynamical behaviors of the following circuit equations of coupled BVP oscillators with hard characteristics

$$\begin{cases} L\frac{di_1}{dt} = v_1 - ri_1, \\ C\frac{dv_1}{dt} = -i_1 - g(v_1) - i_3, \\ L\frac{di_2}{dt} = v_2 - ri_2, \\ C\frac{dv_2}{t} = -i_2 - g(v_2) + i_3, \\ L_0\frac{di_3}{dt} = v_1 - v_2 - R_0 i_3, \end{cases} \tag{1.1}$$

where i_1, i_2, i_3, v_1, and v_2 represent the currents and voltages from the circuit shown in Fig. 1 and L, C, L_0, R_0 and r are positive constants. The function $g(v)$ is the nonlinear conductance which has a fifth order characteristics such that

$$g(v) = d_1 v + d_3 v^3 + d_5 v^5.$$

System (1.1) is constructed by Kitajima and Kawakami [2] through combining two identical oscillators with fifth order characteristics coupled by an inductor containing resistive component. After normalization, the above system becomes

$$\begin{cases} \frac{dx_1}{dt} = \omega y_1 - \sigma x_1, \\ \frac{dy_1}{dt} = -\alpha y_1 - \beta y_1^3 - \gamma y_1^5 - \omega x_1 - \omega_0 x_3, \\ \frac{dx_2}{dt} = \omega y_2 - \sigma x_2, \\ \frac{dy_2}{t} = -\alpha y_2 - \beta y_2^3 - \gamma y_2^5 - \omega x_2 + \omega_0 x_3, \\ \frac{dx_3}{dt} = -\sigma_0 x_3 + \omega_0 (y_1 - y_2), \end{cases} \tag{1.2}$$

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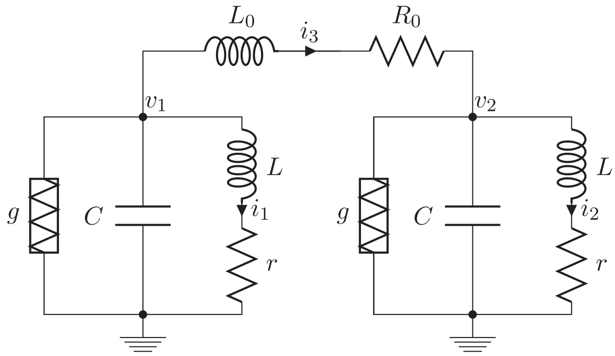

Fig. 1. The circuit.

where

$$y_k = \sqrt{C}v_k, \quad x_k = \sqrt{L}i_k, \quad (k = 1, 2), \quad x_3 = \sqrt{L_0}i_3, \quad \omega = \frac{1}{\sqrt{LC}},$$
$$\omega_0 = \frac{1}{\sqrt{L_0 C}}, \quad \sigma = \frac{r}{L}, \quad \sigma_0 = \frac{R_0}{L_0}, \quad \alpha = \frac{d_1}{C}, \quad \beta = \frac{d_3}{C^2}, \quad \gamma = \frac{d_5}{C^3}.$$

Clearly, $\omega, \omega_0, \sigma, \sigma_0$ are positive constants and α, β, γ are constants.

Kitajima and Kawakami [2] studied System (1.2) and obtained a bifurcation diagram. By choosing α and ω_0 as bifurcation parameters and fixing other parameters $\beta = -1.4,\ \gamma = 0.4,\ \sigma_0 = 0.5,\ \sigma = 0.5$ and $\omega = 0.5$ or 1, they observed some interesting bifurcating behaviors such as stable in-phase, anti-phase and almost in-phase solutions through some numerical simulations. They also observed chaotic attractors caused by successive period-doubling bifurcations of asymmetrical periodic solutions.

However, their study for this system is incomplete and there is basically no rigorous mathematical analysis for the system. They did not give any condition or perform any analysis to indicate how bifurcation parameters depend on other parameters and how to obtain those bifurcating behaviors mentioned above. In this manuscript, we try to carry out a detailed analysis for System (1.2). The explicit conditions are obtained for the system to demonstrate Hopf, BT (Bogdanov–Takens), double Hopf, and fold-Hopf bifurcations, respectively. Also their corresponding normal forms are derived to attain bifurcation diagrams such as Hopf, homoclinic, double limit cycle, and torus bifurcations.

The rest of this manuscript is organized as follows. In Section 2, a detailed presentation is given for the distribution of eigenvalues of the linear part of System (1.2) at (0,0,0,0,0). In Section 3, the first Lyapunov coefficient is computed to perform Hopf bifurcation and hence obtain the stability and direction of periodic solutions. In Section 4, the normal form for BT bifurcation is derived to obtain bifurcation diagrams such as Hopf, homoclinic, and double limit cycle bifurcations. In Section 5, the normal form for double Hopf bifurcation is derived to obtain bifurcation diagrams such as torus bifurcation. In Section 6, the normal form for Fold-Hopf bifurcation is derived to obtain bifurcation diagrams for Fold-Hopf bifurcation. Also numerical simulations are presented to confirm the theoretical results. Finally, we provide a brief discussion in Section 7.

2. Distribution of eigenvalues

Obviously, the point $(0, 0, 0, 0, 0)$ is an equilibrium point of System (1.2) and the linear part of the system at $(0, 0, 0, 0, 0)$ is

$$\begin{cases} \frac{dx_1}{dt} = \omega y_1 - \sigma x_1, \\ \frac{dy_1}{t} = -\alpha y_1 - \omega x_1 - \omega_0 x_3, \\ \frac{dx_2}{dt} = \omega y_2 - \sigma x_2, \\ \frac{dy_2}{t} = -\alpha y_2 - \omega x_2 + \omega_0 x_3, \\ \frac{dx_3}{dt} = -\sigma_0 x_3 + \omega_0 (y_1 - y_2). \end{cases} \tag{2.1}$$

Let

$$A = \begin{pmatrix} -\sigma & \omega & 0 & 0 & 0 \\ -\omega & -\alpha & 0 & 0 & -\omega_0 \\ 0 & 0 & -\sigma & \omega & 0 \\ 0 & 0 & -\omega & -\alpha & \omega_0 \\ 0 & \omega_0 & 0 & -\omega_0 & -\sigma_0 \end{pmatrix}.$$