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New robust stability results for bidirectional associative memory neural networks with multiple time delays

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A B S T R A C T

In this paper, the robust stability problem is investigated for a class of bidirectional associative memory (BAM) neural networks with multiple time delays. By employing suitable Lyapunov functionals and using the upper bound norm for the interconnection matrices of the neural network system, some novel sufficient conditions ensuring the existence, uniqueness and global robust stability of the equilibrium point are derived. The obtained results impose constraint conditions on the system parameters of neural network independent of the delay parameters. Some numerical examples and simulation results are given to demonstrate the applicability and effectiveness of our results, and to compare the results with previous robust stability results derived in the literature.

1. Introduction

In recent years, neural networks have received considerable attention because of their successful applications in image processing, associative memories, optimization problems and other engineering areas [1,2]. Such applications rely on the qualitative stability properties of the designed neural network. Therefore, stability analysis of neural networks plays an important role in the designs and applications of neural networks. On the other hand, time delays occur in VLSI implementation of neural networks due to the finite switching speed of neuron amplifiers, and the finite speed of signal propagation. It is also known that the working with the delayed version of neural networks is important for solving some classes of motion-related optimization problems. However, it has been revealed that time delays may cause instability and oscillation of neural networks. For these reasons, it is of great importance to study the equilibrium and stability properties of neural networks in the presence of time delays. Some results concerning the dynamical behavior of various neural networks with or without delay has been reported in [3–20] and the references therein. We should also point out that, in hardware implementation of neural networks, the network parameters of the system may subject to some changes due to the tolerances of electronic components employed in the design. In such cases, it is desired that the stability properties of neural network should not be affected by the small deviations in the values of the parameters. In other words, the neural network must be globally robustly stable. Global robust stability of standard neural network models with time delays has been studied by many researchers and some important robust stability results have been reported in [21–26].

Bidirectional associative memory (BAM) neural networks were first introduced by Kosko [27,28]. A BAM neural network is composed of neurons arranged in two layers. The neurons in one layer are fully interconnected to the neurons in the other

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layer, while there are no interconnection among neurons in the same layer. It uses the forward and backward information flow to produce an associative search for stored stimulus–response association. One beneficial characteristic of the BAM is its ability to recall stored pattern pairs in the presence of noise. One may refer to [29] for detailed memory architecture and examples of BAM neural networks. This class of networks has successful application perspective in the field of pattern recognition and artificial intelligence due to its generalization of the single-layer auto-associative Hebbian correlator to a two-layer pattern-matched heteroassociative circuit [30]. Some of these applications require that there should be a well-defined computable solution for all possible initial states. From a mathematical point of view, this means that the equilibrium point of the designed neural network is globally asymptotically stable (GAS). The stability of the BAM neural networks has been extensively studied in the literature in the recent years and many different sufficient conditions ensuring the stability of BAM neural networks have been given in [31–43]. However, many of the existing stability results derived for the BAM neural networks can be applicable when only a pure delayed neural network model is employed. In recently published papers [44–48], a hybrid BAM neural network model in which both instantaneous and delayed signaling occur was considered.

In this paper, we study the equilibrium and robust stability properties of hybrid bidirectional associative memory neural networks with multiple time delays. By employing more general types of suitable Lyapunov–Krasovskii functionals and using the upper bound norm for the interconnection matrices of the neural system we obtain some novel delay-independent sufficient conditions for the existence, uniqueness and global robust asymptotic stability of the equilibrium point for hybrid, BAM neural networks with time delays. Some numerical examples are also given to prove that our conditions can be considered as the alternative results to the previous stability results derived in the literature.

2. Model description

Dynamical behavior of a hybrid BAM neural network with constant time delays is described by the following set of differential equations [47]:

$$\begin{aligned} \dot{u}_i(t) &= -a_i u_i(t) + \sum_{j=1}^{m} w_{ji} g_j(z_j(t)) + \sum_{j=1}^{m} w^{\tau}_{ji} g_j(z_j(t-\tau_{ji})) + I_i, \quad \forall i, \\ \dot{z}_j(t) &= -b_j z_j(t) + \sum_{i=1}^{n} v_{ij} g_i(u_i(t)) + \sum_{i=1}^{n} v^{\tau}_{ij} g_i(u_i(t-\sigma_{ij})) + J_j, \quad \forall j, \end{aligned} \tag{1}$$

The BAM neural network model (1) can be regarded as a neural network model having two layers. n denotes number of the neurons in the first layer and m denotes the number of neurons in the second layer. $u_i(t)$ is the state of the ith neuron in the first layer and $z_j(t)$ is the state of the jth neuron in the second layer. a_i and b_j denote the neuron charging time constants and passive decay rates, respectively; $w_{ji},\ w^{\tau}_{ji},\ v_{ij}$ and v^{τ}_{ij} are synaptic connection strengths; g_i and g_j represent the activation functions of the neurons and the propagational signal functions, respectively; and I_i and J_j are the exogenous inputs.

It will be assumed that $a_i, b_j,\ w_{ji},\ w^{\tau}_{ji},\ v_{ij},\ v^{\tau}_{ij},\ \tau_{ji}$ and σ_{ij} in system (1) are uncertain but bounded, and belong to the following intervals:

$$\begin{aligned} &A_I := \{A = diag(a_i) : 0 < \underline{A} \leqslant A \leqslant \overline{A}, i.e., 0 < \underline{a}_i \leqslant a_i \leqslant \overline{a}_i,\ i = 1,2,\ldots,n,\ \forall A \in A_I\}, \\ &B_I := \{B = diag(b_j) : 0 < \underline{B} \leqslant B \leqslant \overline{B}, i.e., 0 < \underline{b}_j \leqslant b_j \leqslant \overline{b}_j,\ j = 1,2,\ldots,m,\ \forall B \in B_I\}, \\ &W_I := \{W = (w_{ji})_{m\times n} : \underline{W} \leqslant W \leqslant \overline{W}, i.e., \underline{w}_{ji} \leqslant w_{ji} \leqslant \overline{w}_{ji},\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m,\ \forall W \in W_I\}, \\ &V_I := \{V = (v_{ij})_{n\times m} : \underline{V} \leqslant V \leqslant \overline{V}, i.e., \underline{v}_{ij} \leqslant v_{ij} \leqslant \overline{v}_{ij},\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m,\ \forall V \in V_I\}, \\ &W^{\tau}_I := \{W^{\tau} = (w^{\tau}_{ji})_{m\times n} : \underline{W^{\tau}} \leqslant W \leqslant \overline{W}^{\tau}, i.e., \underline{w}^{\tau}_{ji} \leqslant w^{\tau}_{ji} \leqslant \overline{w}^{\tau}_{ji},\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m,\ \forall W^{\tau} \in W^{\tau}_I\}, \\ &V^{\tau}_I := \{V^{\tau} = (v^{\tau}_{ij})_{n\times m} : \underline{V}^{\tau} \leqslant V \leqslant \overline{V}^{\tau}, i.e., \underline{v}^{\tau}_{ij} \leqslant v^{\tau}_{ij} \leqslant \overline{v}^{\tau}_{ij},\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m,\ \forall V^{\tau} \in V^{\tau}_I\}, \\ &\tau_I := \{\tau = (\tau_{ji})_{m\times n} : \underline{\tau} \leqslant \tau \leqslant \overline{\tau}, i.e., \underline{\tau}_{ji} \leqslant \tau_{ji} \leqslant \overline{\tau}_{ji},\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m,\ \forall \tau \in \tau_I\}, \\ &\sigma_I := \{\sigma = (\sigma_{ij})_{n\times m} : \underline{\sigma} \leqslant \sigma \leqslant \overline{\sigma}, i.e., \underline{\sigma}_{ij} \leqslant \sigma_{ij} \leqslant \overline{\sigma}_{ij},\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m,\ \forall \sigma \in \sigma_I\}. \end{aligned} \tag{2}$$

In order to establish the desired stability properties of neural network model (1), it is first necessary to specify the class of activation functions. The activation functions we employ in (1) are assumed to satisfy the following conditions:

(H1) There exist some positive constants $\ell_i,\ i = 1,2,\ldots,n$ and $k_j,\ j = 1,2,\ldots,m$ such that

$$0 \leqslant \frac{g_i(\bar{x}) - g_i(\bar{y})}{\bar{x} - \bar{y}} \leqslant \ell_i, \quad 0 \leqslant \frac{g_j(\hat{x}) - g_j(\hat{y})}{\hat{x} - \hat{y}} \leqslant k_j$$

for all $\bar{x}, \bar{y}, \hat{x}, \hat{y} \in R$. This class of functions will be denoted by $g \in \mathcal{K}$.

(H2) There exist positive constants $M_i,\ i = 1,2,\ldots,n$ and $L_j,\ j = 1,2,\ldots,m$ such that $|g_i(u)| \leqslant M_i$ and $|g_j(z)| \leqslant L_j$ for all $u, z \in R$. Note that this assumption implies that the activation functions are bounded and this class of functions will be denoted by $g \in \mathcal{B}$.