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Modeling the effect of time delay in controlling the carrier dependent infectious disease – Cholera

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ABSTRACT

A delay mathematical model for the control of cholera epidemic is proposed and analyzed. It is assumed that the disease spreads through carriers, which makes the human food contaminated by transporting bacteria from the environment. It is also assumed that insecticides are used to control the carriers with the rate proportional to the density of carriers. The analysis of model shows that the disease may be controlled by spraying insecticides but a longer delay in spraying insecticides may destabilize the system. Simulation is also carried out to support the analytical results.

1. Introduction

Cholera is an acute infectious disease in which intestine gets infected by the bacterium *Vibrio cholerae* (*V. cholerae*). Generally the infection is mild but sometimes it may be severe and if not treated properly, death may occur within hours. The period of infection of cholera ranges from few hours to 5 days but in general it is of 1–2 days. The main symptoms of cholera are watery diarrhea, vomiting, rapid dehydration, metabolic acidosis, and hypovolemic shock. The world have acquainted and feared cholera for hundreds of years. Several cholera epidemics have occurred worldwide during the 15th–18th centuries. During the 19th and 20th centuries, seven cholera pandemics have ravaged the humankind [1,2]. The seventh cholera pandemic originated from Indonesia in 1961 and thereafter spread around the globe. During the seventh pandemic in 1991, India has been recognized as an endemic zone for cholera due to *V. cholerae* O1 serogroup Ogawa, biotype El Tor, and serogroup O139. In 2005, an outbreak of cholera, dominated by *V. cholerae* O1 Inaba, has also been reported in India [2,3]. The disease is still common in several parts of Asia and Africa.

Prevalence of cholera is closely related to poor environmental conditions and lack of basic infrastructure in developing countries. In earlier times, the main mode of transmission of cholera infection was believed to be the consumption of contaminated water. Therefore, government of various countries made effectual efforts to make drinking water safe for use by means of treatments such as filtration and chlorination. But it is observed that even after the treatment of water, the disease is prevalent in some areas. Some studies reveal that the common housefly, or flies in general, can serve as mechanical vectors of numerous kind of pathogens such as bacteria [4,5], protozoa [6], and helminth eggs [7]. It suggests that, pathogen of cholera may also spread via flies. The increased incidence of dysentery during periods of high density of flies has been reported from various places [5,8,9]. It fortifies the notion that flies are also major transporter of cholera-causing bacteria. Basically,

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the flies transport the bacteria responsible for cholera disease to the human food and make it contaminated [10,11]. Due to the uptake of this contaminated food, people acquire cholera infection [1,12].

In addition to cholera, there are diseases like typhoid fever and other enteric diseases, which spread in human population through carriers. Most of the developing countries are affected by such diseases due to lack of sanitation and wide occurrence of carriers. Various studies have been conducted to understand the role of carriers (e.g., flies, ticks, mites, etc.) and bacteria in the spread of infectious diseases [12–27]. In particular, Hethcote [19] studied a nonlinear mathematical model for communicable diseases by assuming that the carrier population is constant. But, in general the carrier population is not constant and it depends on various environmental factors, like humidity, temperature, etc. Some human-related activities like discharge of household wastes, open sewage drainage, industrial effluents in residential areas, open water storage tanks, ponds, etc., also enhance the growth of carriers. The role of human-related activities on the growth of carrier population has been studied explicitly by some modelers [20–23,25]. Singh et al. [23] proposed and analyzed SIS and SIRS mathematical models for the spread of carrier dependent infectious diseases by assuming that the growth rate and carrying capacity of carrier population are increasing function of total human population density. They have shown that the number of infectives increases as the human-related activities, which makes the environment more conducive for the growth of carriers, increase. Codeco [24] also studied a mathematical model for cholera epidemic by incorporating the role of environmental reservoirs. In this study, the minimum number of conditions for the development of endemicity of cholera have been derived and the persistence of cholera is also discussed. For cholera epidemic it is found that by controlling the environmental fluctuations the severity of the disease can be managed [25].

Although commendable studies have been conducted to explore the role of carriers in the spread of infectious diseases, a little attention is paid to their control. An efficient way to control the spread of carrier dependent infectious diseases may be vaccination of susceptibles [27]. As the spread of diarrheal diseases is closely related to the seasonal increase in abundance of flies, their control helps in declining the cases of enteric diseases. Therefore, generally government uses insecticides, in particular DDT, to control the flies in the environment during cholera outbreaks. But the excessive use of insecticides is harmful for plants and human health, so some people called ‘trained squads’ are especially trained to perform the fly control in a region. These trained squads are employed by the government to spray DDT in houses as well as in other relevant areas and the amount of insecticides used in fly control depends on the density of carriers [28].

As discussed above, the control of carrier population may be an effective tool for controlling the spread of carrier dependent infectious diseases, especially cholera. So we have concentrated our study on spread of cholera disease due to consumption of food contaminated by carriers present in the environment. Therefore, in this paper, we propose and analyze a mathematical model for the control of cholera disease by controlling the density of carrier population using insecticides. The density of carrier population known to the trained squads may be few days old, thus the rate of change of concentration of insecticides is assumed to be proportional to that density of carrier population, which leads for incorporation of time delay in the rate of spray of insecticides. Furthermore, some amount of insecticides may washout with the passage of time and some may be consumed by the carriers. Hence, the depletion of insecticides has also been considered in the modeling process.

2. Mathematical model

Cholera does not spread directly from one person to another, therefore the casual contacts with an infected person is not a risk for getting infection [29]. Hence, in the modeling process, it is assumed that susceptibles get infected due to the presence of carriers, which contaminate the food. It is also assumed that the carriers grow logistically in their natural environment and human-related activities further enhance their growth rate. We consider that the growth rate of carriers is controlled by spraying insecticides with a rate proportional to the density of carriers, τ time before.

Let $N(t)$ be the total human population at any time t, which is divided into two subclasses, namely (i) susceptible class $X(t)$ and (ii) infective class $Y(t)$. Let $C(t)$ and $C_h(t)$ represent the density of carrier population and the concentration of insecticides respectively at time t. It is assumed that the rate of introduction of insecticides is proportional to $C(t-\tau)$ and its natural depletion rate is proportional to its concentration. It is also assumed that the depletion rate of insecticides due to its uptake by carriers is proportional to the density of carriers as well as the concentration of insecticides. The decline rate of carrier population due to uptake of insecticides is also taken to be proportional to the density of carriers and the concentration of insecticides.

With the above considerations, the model dynamics is governed by the following system of nonlinear delay differential equations:

$$\begin{aligned}
\frac{dX(t)}{dt} &= A - \lambda X(t)C(t) - dX(t) + \nu Y(t),\\
\frac{dY(t)}{dt} &= \lambda X(t)C(t) - (\nu+\alpha+d)Y(t),\\
\frac{dC(t)}{dt} &= rC(t)\left(1-\frac{C(t)}{K}\right) - s_1C(t) + r_1N(t)C(t) - \theta_2C(t)C_h(t),\\
\frac{dC_h(t)}{dt} &= \theta C(t-\tau) - \theta_0C_h(t) - \theta_1C(t)C_h(t),
\end{aligned} \tag{1}$$