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Optimal control of computer virus under a delayed model

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ABSTRACT

This paper addresses the issue of how to suppress the spread of computer virus by means of the optimal control method. First, a controlled delayed computer virus spread model is established. Second, an optimal control problem is formulated by making a tradeoff between the control cost and the control effect. Third, the optimal control strategies are theoretically investigated. Finally, it is experimentally shown that the spread of infected nodes can be suppressed effectively by adopting an optimal control strategy.

1. Introduction

Malicious computer viruses can damage computer systems (nodes, for short) where they reside by erasing data, breaking files, or modifying the normal operation. Indeed, enormous existing computer viruses have formed a high hardness to today's corporations as well as individuals. Developing mathematical models governing the propagation of computer virus helps understand the behavior of computer virus and, on this basis, stop its spread. Due to the high similarity between computer virus and biological virus, various computer virus propagation models were proposed [1–7]. Furthermore, due to the inevitable delay from the time a computer system is infected with a virus to the time the virus exhibits its effect, a realistic model should incorporate the delay factor [8,9]. Although having been widely applied to the control of spread of epidemics [10–12], the optimal control theory, to our knowledge, has yet to be considered in the context of computer virus.

This paper addresses the issue of controlling the spread of computer virus by means of the optimal control method. First, a delayed computer virus spread model with control is established by introducing an appropriate control variable into a delayed model presented in [9]. Second, an optimal control problem is described by suggesting an objective functional for optimizing the control cost and effect. Third, the existence and uniqueness of an optimal control strategy are proved, and a necessary condition for an optimal control strategy is presented. Finally, numerical examples demonstrate that the spread of computer virus can be controlled effectively by use of an optimal control strategy.

This paper is organized as follows. Section 2 formulates the model and the corresponding optimal control problem, Section 3 conducts a theoretical study of the optimal control strategy, Section 4 gives several numerical examples, and Section 5 concludes this work.

2. The model and optimal control problem

Our work is based on the delayed model for computer virus spread presented by Han and Tan [9], which assumes that the individuals in a computer system can switch among the susceptible, infected, and recovered states according to $S \to I \to R \to S$. The model is formulated as the following system of delayed differential equations:

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$$\begin{cases} \frac{dS}{dt} = (1-p)b - \mu S - \beta S(t-\tau_1)I(t-\tau_1) + \nu R(t-\tau_2), \\ \frac{dI}{dt} = \beta S(t-\tau_1)I(t-\tau_1) - (\mu+\gamma+\alpha)I, \\ \frac{dR}{dt} = pb + \gamma I - \mu R - \nu R(t-\tau_2), \end{cases} \tag{1}$$

where b is the number of newly coming nodes, p is the immune rate of a newly coming node, β is the infection rate of an infected node, μ is the natural leaving rate of a node, ν is the loss rate of immunity of a recovered node, γ is the recovery rate of an infected node, α is the leaving rate of an infected node due to the action of the virus, τ_1 is the latent period of an infected node, and τ_2 is the immune period of a recovered node [9].

For our purpose, let $[0, T]$ denote the time period during which a control strategy is imposed on system (1), and introduce a Lebesgue square integrable control function $u(t)$ $(0 \leqslant t \leqslant T)$, the budget for buying antivirus software at time t, which is normalized to fall between 0 and 1. Then the admissible set of control functions is given by

$$U = \left\{ u(t) \in L^2[0,T] : 0 \leqslant u(t) \leqslant 1,\ 0 \leqslant t \leqslant T \right\}.$$

To introduce the controlled model, the following assumption is imposed.

(A) Under the action of $u(t)$, there are $\omega u(t)I(t)$ infected nodes that would become susceptible at time t, whereas there are $(1-\omega)u(t)I(t)$ infected nodes that would become recovered at time t, where $\omega \in [0, 1]$.

The corresponding model for the controlled propagation of computer virus then follows as

$$\begin{cases} \frac{dS}{dt} = (1-p)b - \mu S - \beta S(t-\tau_1)I(t-\tau_1) + \nu R(t-\tau_2) + \omega u(t)I, \\ \frac{dI}{dt} = \beta S(t-\tau_1)I(t-\tau_1) - (\mu+\gamma+\alpha)I - u(t)I, \\ \frac{dR}{dt} = pb + \gamma I - \mu R - \nu R(t-\tau_2) + (1-\omega)u(t)I, \end{cases} \tag{2}$$

with initial conditions $S(0) = S_0, I(0) = I_0$, and $R(0) = R_0$.

Our objective is to find a control function $u(t)$ so that (a) the accumulated number of infected nodes during the time period $[0, T]$ is minimized, and (b) the total budget for buying antivirus software during the same period is also minimized. As a tradeoff between these two requirements, we shall try to find a control function $u(t)$ so that the functional

$$J(u) = \int_0^T \left[I(t) + \frac{\epsilon u^2(t)}{2} \right] dt \tag{3}$$

is minimized, where ϵ is a tradeoff factor.

For the controlled system (2), the basic reproduction number, which is defined as the average number of secondary infections produced when one infected node is introduced into a host system fully composed of susceptible nodes, is determined by

$$R_p(t) = \frac{\beta b[\mu(1-p)+\nu]}{\mu(\mu+\nu)[\mu+\alpha+\gamma+u(t)]}.$$

Clearly, the value of $R_p(t)$ would drop with increasing $u(t)$. For quite a number of epidemiology models, an infection can get started in a fully susceptible population if and only if $R_p > 1$. For the controlled system (2) with time delays $\tau_1 = 0$ and $\tau_2 = 0$, there always exists a disease-free equilibria

$$E_0 = \left(\frac{\mu b(1-p)+\nu}{\mu(\mu+\nu)}, 0, \frac{pb}{\mu+\nu} \right).$$

If $R_p(t) \leqslant 1$ or, equivalently, $u(t) \geqslant \frac{\beta b[\mu(1-p)+\nu]}{\mu(\mu+\nu)} - (\mu+\alpha+\gamma)$, the infection in the community would die out. Whereas if $R_p(t) > 1$ or, equivalently, $u(t) < \frac{\beta b[\mu(1-p)+\nu]}{\mu(\mu+\nu)} - (\mu+\alpha+\gamma)$, there would exist a unique positive epidemic equilibria $E_1 = (S_1(t), I_1(t), R_1(t))$, where

$$\begin{cases} S_1(t) = \dfrac{\mu+\alpha+\gamma+u(t)}{\beta}, \\ I_1(t) = \dfrac{\mu(\mu+\nu)[\mu+\alpha+\gamma+u(t)]}{\beta[(\mu+\alpha)(\mu+\nu)+\mu\gamma+\mu(1-\omega)u(t)]}(R_p(t)-1), \\ R_1(t) = \dfrac{pb+\gamma I_1(t)}{\mu+\nu}. \end{cases}$$

3. Properties concerning optimal control strategies

Clearly, the optimal control problem (2) and (3) has the Lagrangian