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Status of the differential transformation method

C. Bervillier

Laboratoire de Mathématiques et Physique Théorique, UMR 7350 (CNRS), Fédération Denis Poisson, Université François Rabelais, Parc de Grandmont, 37200 Tours, France

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ABSTRACT

Further to a recent controversy on whether the differential transformation method (DTM) for solving a differential equation is purely and solely the traditional Taylor series method, it is emphasized that the DTM is currently used, often only, as a technique for (analytically) calculating the power series of the solution (in terms of the initial value parameters). Sometimes, a piecewise analytic continuation process is implemented either in a numerical routine (e.g., within a shooting method) or in a semi-analytical procedure (e.g., to solve a boundary value problem). Emphasized also is the fact that, at the time of its invention, the currently-used basic ingredients of the DTM (that transform a differential equation into a difference equation of same order that is iteratively solvable) were already known for a long time by the "traditional"-Taylor-method users (notably in the elaboration of software packages – numerical routines – for automatically solving ordinary differential equations). At now, the defenders of the DTM still ignore the, though much better developed, studies of the "traditional"-Taylor-method users who, in turn, seem to ignore similarly the existence of the DTM. The DTM has been given an apparent strong formalization (set on the same footing as the Fourier, Laplace or Mellin transformations). Though often used trivially, it is easily attainable and easily adaptable to different kinds of differentiation procedures. That has made it very attractive. Hence applications to various problems of the Taylor method, and more generally of the power series method (including noninteger powers) has been sketched. It seems that its potential has not been exploited as it could be. After a discussion on the reasons of the "misunderstandings" which have caused the controversy, the preceding topics are concretely illustrated. It is concluded that, for the sake of clarity, when the DTM is applied to ODEs, it should be mentioned that the DTM exactly coincides with the traditional Taylor method, contrary to what is currently written.

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1. Introduction

The differential transformation method (DTM) of Pukhov [1–5] and Zhou [6] is frequently presented as a (relatively) new method for solving differential equations.[1] Though based on Taylor series, it would be different from the traditional Taylor (or power) series method presented in usual textbooks as e.g., [7]. This distinction has been the object of a recent dispute [8,9]. Independently of whether this distinction was present in the original ideas,[2] it is at least seemingly clearly expressed and often repeated since the second half of the 1990s when the DTM has more systematically been used "to solve differential equations"

E-mail address: claude.bervillier@lmpt.univ-tours.fr

[1] An extensive presentation of the DTM is given in Section 2.2.

[2] The most frequently cited original works on DTM are inaccessible to me because they are written either in Russian [4,5] or in Chinese [6]. Except those two references, I have systematically solely cited articles written in English and, hopefully, accessible to all.

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[10–16]. For example, in [14] the "*differential transformation technique*", is presented"*as an extended Taylor series method*" and in [15], where the differential equation referred to is

$$\frac{dx}{dt} = f(x,t), \tag{1}$$

one can read (in addition):

"*The differential transformation technique is one of the numerical methods for ordinary differential equations. It uses the form of polynomials as the approximation to exact solutions which are sufficiently differentiable. This is in contrast to the traditional high-order Taylor series method, which requires the computation of the necessary derivatives of $f(x,t)$ and is computationally intensive as the order becomes large. Instead, the differential transformation technique provides an iterative procedure to get the high-order Taylor series. Therefore, it can be applied to the case with high order.*" [15, p. 25] (see also [16]).

That statement is important since then the idea that the "traditional" Taylor series method would require the explicit calculation of high-order derivatives of $f(x,t)$ is repeated in many articles on the DTM as if it would be a convincing reason to make a definitive distinction between the two methods.

Before going any further, it is worth recalling what is the Taylor series method[3]:

Definition 1 (*(Formal or raw) Taylor series method*). The Taylor series method consists in expressing the solution of (1) as a power series expansion about the initial time t_0:

$$\left.\begin{array}{l} x(t) = \sum\limits_{k=0}^{\infty}(t-t_0)^k \frac{1}{k!}\frac{d^kx}{dt^k}\Big|_{t=t_0}, \\ x(t_0) = \alpha_0 \end{array}\right\} \tag{2}$$

in which the derivatives $\frac{d^kx}{dt^k}\Big|_{t=t_0}$ are such that (1) is satisfied order by order in powers of $(t-t_0)$. As consequence, the Taylor coefficients of the expansion are completely determined once the initial parameter α_0 is fixed.

Notice that this definition specifies neither the way the derivatives $\frac{d^kx}{dt^k}\Big|_{t=t_0}$ are calculated nor the convergent property of the series. Thus it is surely clear to all that the *formal* Taylor series method (described in textbooks such as [7]) requires only that the series of $x(t)$ be obtained by any means whatsoever. Consequently, it is obvious that if the DTM is a "*procedure to get the high-order Taylor series*" [15,16], then the method used to solve the differential equation is not the DTM but the Taylor series method (at least formally). Consequently to really appreciate the above quotation of [15], it is necessary to understand what actually is the "traditional" Taylor series method. No doubt that for the DTM users, the naive explicit computation of high order derivatives of $f(x,t)$ is an integral part of the "traditional" method. One of the object of the present article is to show that this allegation is false.

In fact, as one may anticipate from the presence of the terms "*technique* " and "*numerical method*", the Taylor series method mentioned in the above quotation of [15], though not explicitly defined, is not the *formal* one.[4] It would be possible that it is a problem of semantics which has generated misunderstanding.

Those words (technique, numerical) have wittingly been used to qualify the DTM. Indeed, some authors would like to see the DTM not as a formal method, but exclusively as a new *numerical* approach:

"*…the construction of power-series solutions has been generally thought of as an analytic tool and not as the basis for numerical algorithms. This is changing, and algorithmically constructing power-series solutions to ordinary differential equations is gaining in popularity. This is now often called the Differential Transform (ation) Method (DTM).*" [17].

However, if this view corresponds well to Pukhov's initial aim of "*consider*[ing] *the problem of feasibility of constructing computer-specialized procedures oriented toward automatic solution of Taylor equations*" [1], the actual use of power-series solution as the basis for numerical algorithms is not at all new, see, for example, [18–22]. Actually:

Remark 1 (*(Numerical) Taylor series method*). Taylor series may be used as a tool to numerically solve the initial value problem associated with (1). To this end the convergence of the Taylor series (2) must be controlled. The most frequently used procedure is the stepwise (or piecewise) procedure described in Section 2.1.2.1.

In this respect of the numerical treatment of ODEs, and considering the high level of numerical development of the traditional Taylor method (see Section 2.1.2.3), one is forced to acknowledge that the current use of the DTM cannot be seen as a real technological break-through (despite the huge number of publications on the subject). One may easily verify that, the DTM (as it is currently used) brings no new capability into the numerical treatment of (at least ordinary) differential equations (see Section 2). Consequently, to possibly distinguish it from the traditional Taylor series method only remains the way the expansion coefficients are calculated, in apparent accordance with the above quotation of [15].

Unfortunately, when looking at the "old papers" using the Taylor method, in that respect of calculating the terms of the Taylor expansion, one quickly realizes that the basical tricks of the DTM (that transform the differential equation into an

[3] For the sake of a convenient writing, extensive descriptions of the "traditional" Taylor series method and of the DTM are postponed to Section 2.

[4] Which is the best known formulation of the Taylor method.

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