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Three-layer supply chain – A production-inventory model for reworkable items

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ABSTRACT

This article develops an integrated production-inventory model considering perfect and imperfect quality items, product reliability and reworking of defective items in the environment of supply chain management. Manufacturer, supplier and retailer are the members of the supply chain where supplier delivers raw materials to the manufacturer and sends back the defective raw materials after completion of inspection at a single lot with less market price. In this system, production starts to produce good items at the beginning of the production. The production system may undergo an "out-of-control" state from an "in-control" state, after a certain time that follows a probability density function. The density function varies with reliability of the machinery system that may be controlled by new technologies, investing more capital. The defective items produced in "out-of-control" state are reworked at a cost just after the regular production time. This model considers the impact of business strategies such as optimal order size of raw materials, production rate and unit production cost, and idle times in different sectors in a collaborating marketing system. An analytical method is employed to optimize the production rate and raw material order size for maximum expected average profit. A numerical example along with four graphical illustrations is considered and its sensitivity analysis is provided to test feasibility of the model.

1. Introduction

In the study of supply chain network, mathematical modeling, analysis and computation have been an active area of research. The complexity of the relationships among the various decision-makers, such as suppliers, manufacturers, distributors and retailers as well as the practical importance of the topic (manufacturing, transportation and logistics, retailing/marketing, cost, quality, reliability, delivery, flexibility, etc.) for the efficient movement of products are interconnected and relevant in a oligopolistic marketing system.

Rosenblatt and Lee [19] studied a model (RL model) where the probability distribution of the time of shifting from in-control state to out-of-control state follows an exponential distribution. They assumed that the defective items produced in out-of-control state could be reworked instantaneously at a cost and found that the presence of defective products results in smaller lot size. On the basis of RL model, Lee and Rosenblatt [10] showed that process inspection during the production-run time can detect the shifting time and it could be restored earlier. Liu and Yang [11] investigated a single stage production system with imperfect process of delivering of reworkable and non-reworkable items. The effect of defective items on the lot

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sizing policy was noted in the works of Sana et al. [28,29], among others. Kim and Hong [6] assumed that defective items are detected after the production run and the defective items were reworked at a cost. Salameh and Jaber [20] extended the traditional EPQ/EOQ model by accounting for imperfect quality items when using the EPQ/EOQ formulae. Also, they assumed that the poor quality items are sold in a single batch at the end of the total 100% screening process. Cardenas-Barron [14] had written a note on the paper of Salameh and Jaber's [20] where they corrected an equation and showed that the error only affects the optimum value of the order size. Goyal and Cardenas-Barron [23] extended Salameh and Jaber's [20] model and proposed a practical approach to determine EPQ for items with imperfect quality. Eroglu and Ozdemir [2] extended Salameh and Jaber [20] model independently, allowing shortages. Chung and Hou [13] also developed an inventory model to determine an optimal run time for a deteriorating production system with shortages. Banerjee and Kim [1] studied the integrated inventory models in which the vendor and the buyer coordinate their production and ordering policies, in order to lower the joint inventory costs. Khouja and Mehrez [18] assumed three coordination mechanisms between the members of the supply chain and showed that some coordination mechanisms lead to significant reduction in total cost. Cardenas-Barron [15] extended the model of Khouja and Mehrez [18] by algebraic method, considering n-stage multi-customer supply chain inventory system. Hill [22] provided a global optimal batching and shipment policy for single vendor and single buyer integrated problem, combining the equal shipment size and policies. Jalbar et al. [4] investigated a multi-echelon inventory system in which one vendor supplies an item to multiple buyers. They formulated the problem in terms of integer-ratio policies and developed a heuristic method. Chiu [32] developed an EPQ model with backorders by combining the assumptions of a proportionate of the defective items are reworked to make them good quality items instead of reworking on all of the defective items and the remaining items are sold at a less price. Yang and Wee [7] developed a supply chain model by collaborating producer, distributor and retailer as three members of the chain. They showed that the total cost of independent decision making by each individual entity of the chain is larger significantly than in the integrated approach. Cardenas-Barron [16] presented a simple derivation to find out optimal manufacturing batch size with rework process at single stage production system. Cardenas-Barron [17] developed an EPQ model with planned backorders for determining the production lot size and the size of backorders in an imperfect production process where all defective items were reworked at the same cycle. Sana and Chaudhuri [27], Sana [24,25], Sarkar et al. [3] and Chiu et al. [33] showed that the defective items could be reworked at a cost where overall production-inventory costs could be reduced significantly. Chiu [30] and Inderfruth et al. [12] studied a deterministic planning problem where the new and recovering defective items of the same product were manufactured in the same facility. Chiu et al. [34] discussed a lot size problem with random scrap rate and backlogging by alternative approach instead of calculated method. Chiu, Wang and Chiu [31] extended the work of Chiu [32] and studied the optimal run-time problem of EPQ model with scrap, reworking of defective items and stochastic breakdowns. Lin, Chiu and Ting [9] extended the algebraic approach to the model examined by Chiu [30] and found out the optimal replenishment policy for the expected cost function. Sarker et al. [5] addressed the issue relating to reworking of defective items in a multi-stage production system by considering two operational policies: reworking of defective items within the same cycle and after N cycles. Biswas and Sarker [21] described an inventory system of a single production process with an in-cycle rework policy of scrap with 100% inspection. Recently, Liao et al. [8] investigated maintenance and imperfect process with EPQ model involving a deteriorating production system with an increasing hazard rate. Recently, Sana [26] developed an integrated production-inventory model of perfect and imperfect quality products in a three-layer supply chain.

In this paper, we construct a model of three-layer supply chain containing supplier, manufacturer and retailer where imperfect items with retailers are sent back to the outside supplier at a price and the imperfect items with the manufacturer are reworked at a cost after regular production. The average profit of the supplier is evaluated and the average profit of the manufacturer is calculated when the out-of-control state occurs or does not occur during regular production-run time. Retailer's individual average profit is also obtained. The integrated expected average profit and its difference with the total expected average profit by Stakelberg approach is calculated and a numerical example is considered to test which one is latter.

2. Fundamental assumptions and notation

The following assumptions are made to develop the model:

2.1. Assumptions

(i) Model is developed for a single type of products.
(ii) Demand rates for retailers and customers are considered to be constant.
(iii) Replenishment rate of supplier is instantaneously infinite, but it's size is finite.
(iv) Production rate is a decision variable.
(v) Production cost per unit item varies with production rate.
(vi) Defective items at supplier level follow a probability distribution function.
(vii) No product of manufacturer is scraped at any stage.

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