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The g-good-neighbor conditional diagnosability of hypercube under PMC model ☆,☆☆

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ABSTRACT

Processor fault diagnosis plays an important role in multiprocessor systems for reliable computing, and the diagnosability of many well-known networks has been explored. For example, hypercubes, crossed cubes, möbius cubes, and twisted cubes of dimension n all have diagnosability n. The conditional diagnosability of n-dimensional hypercube Q_n is proved to be $4(n-2)+1$ under the PMC model. In this paper, we study the g-good-neighbor conditional diagnosability of Q_n under the PMC model and show that it is $2^g(n-g)+2^g-1$ for $0 \leqslant g \leqslant n-3$. The g-good-neighbor conditional diagnosability of Q_n is several times larger than the classical diagnosability.

1. Introduction

With the rapid development of technology, the need for high-performance large multiprocessor systems has been continuously increasing day by day. Since all the processors run in parallel, the reliability of each processor in multiprocessor systems becomes an important issue for parallel computing. In order to maintain the reliability of such multiprocessor systems, whenever a processor (node or vertex) is found faulty, it should be replaced by a fault-free processor.

Fault-tolerant computing for the hypercube has been of interest to many researchers. The process of identifying faulty vertices is called the *diagnosis* of the system. System diagnosis can be done in two different approaches, that is, circuit-level diagnosis and system-level diagnosis. In circuit-level diagnosis, the processors must be tested one after one by the human labor, which induces diagnosis complicated and possibly inaccurate. On the other hand, system-level diagnosis could be done automatically by the system itself. Thus, system-level diagnosis appears to be an alternative to circuit-level testing in a large multiprocessor system. Many terms for system-level diagnosis have been defined and various models have been proposed in the literature [2,7,16,20]. If all allowable fault sets can be diagnosed correctly and completely based on a single syndrome, then the diagnosis is referred to as *one-step diagnosis* or *diagnosis without repairs*.

We use the widely adopted PMC model [20] as the fault diagnosis model. In [9], Hakimi and Amin proved that a multiprocessor system is t-*diagnosable* if it is t-connected with at least $2t+1$ vertices. Besides, they gave a necessary and sufficient condition for verifying if a system is t-diagnosable under the PMC model. Recently, Mánik and Gramatová [17,18] propose a

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diagnosis algorithm under the PMC model which use boolean formalization. Fan et al. show the DCC linear congruential graphs, $G(F, 2^p)$, is $2t$-diagnosable where $p \geqslant 3$ and $2 \leqslant t \leqslant p-1$ [6]. Ahlswede and Aydinian study the diagnosability of large multiprocessor networks [1]. The hypercube [15,21] is a well-known interconnection network for multiprocessor systems. Reviewing the previous papers, there are several variations of the hypercube [12], for example, the crossed cube [4], the möbius cube [5], and the twisted cube [10]. For each of these cubes, an n-dimensional cube can be constructed from two copies of $(n-1)$-dimensional cubes by adding a perfect matching between them. One of the common property among them is that all these variations have diagnosability n under the PMC model.

In classical measures of system-level diagnosability for multiprocessor systems, it has generally been assumed that any subset of processors can potentially fail at the same time. If there is a vertex v whose neighboring vertices are faulty simultaneously, there is no way of knowing the faulty or fault-free status of v. As a consequence, the diagnosability of a system is upper bounded by its minimum degree. Motivated by the deficiency of the classical measurement of diagnosability, Lai et al. [13] introduced a measure of *conditional diagnosability* by claiming the property that any faulty set cannot contain all neighbors of any processor. Under this condition, they showed that the conditional diagnosability of the n-dimensional hypercube Q_n is $4(n-2)+1$. We are then led to the following question: how large the maximum value t can be such that a graph G remains t-diagnosable under the condition that every vertex v has at least g fault-free neighboring vertices. More precisely, we assume the faulty set F satisfies the condition that each vertex v in $G-F$ has at least g good neighbors. We notice that, considering the situation that all the neighbors of each vertex cannot fail simultaneously, many properties of the network would be much better, including the connectivity and diagnosability. The aim of this paper is to study more of these better properties.

In this paper, we extend the concept of conditional diagnosability and propose a new measure of diagnosability. We define *g-good-neighbor conditional diagnosability* as the maximum number of faulty vertices that the system can guarantee to identify under the condition that every fault-free vertex has at least g fault-free neighbors. In this paper, we show that the g-good-neighbor conditional diagnosability of Q_n is $2^g(n-g)+2^g-1$ under the PMC model, which is several times larger than the classical diagnosability of Q_n depending on the condition g.

The rest of this paper is organized as follows: Section 2 provides terminology and preliminaries for diagnosing a system. In Section 3, we show the proof of the g-good-neighbor conditional diagnosability of Q_n. Finally, our conclusions are given in Section 4.

2. Preliminaries

2.1. Notations

A multiprocessor system or a network is usually represented as an undirected graph where vertices represent processors and edges represent communication links. Throughout this paper, we follow [11,22] for the graph definitions and notations, and we focus on the undirected graph without loops (simply abbreviated as graph).

Let $G = (V, E)$ be a *graph* where V is a finite set and E is a subset of $\{(u,v)|(u,v)$ is an unordered pair of $V\}$. We say that V is the *vertex set* and E is the *edge set*. We use $n(G) = |V|$ to denote the cardinality of V. The *degree* of a vertex v in G, written as $deg_G(v)$ or $deg(v)$, is the number of edges incident to v. The graph G is *k-regular* if every vertex has degree k. The *neighborhood* of a vertex v in G, written $N_G(v)$ or $N(v)$, is the set of vertices adjacent to v. We use $N(A) = \{x \;|y \in A,\; x \in G-A$, and $(x,y) \in E(G)\}$ to denote the neighborhood of a vertex subset A of G. Two vertices u and v are *adjacent* in G if $(u,v) \in E$. A graph G is *connected* if for any two vertices, there is a path joining them, otherwise it is *disconnected*. For a set S of V, the notation $G-S$ represents the graph obtained by removing the vertices in S from G and deleting those edges with at least one end vertex in S. If $G-S$ is disconnected, then S is called a *separating set* (or a *vertex cut*). A graph H is a *subgraph* of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. A *component* of a graph G is its maximal connected subgraph. The *connectivity* $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left. A graph G is *k-connected* if its connectivity is at least k.

2.2. Diagnosability

Under the classical PMC model [20], adjacent processors are capable of performing tests on each other. For two adjacent vertices u and v in V, the ordered pair (u,v) represents the test performed by u on v. In this situation, u is called the *tester* and v is called the *tested vertex*. The outcome of a test (u,v) is either 1 or 0 with the assumption that the testing result is regarded as reliable if the tester u is fault-free. However, the outcome of a test (u,v) is unreliable, provided that the tester u itself is originally a faulty processor. Suppose that the tester u is fault-free, then the result would be 0 (respectively, 1) if v is fault-free (respectively, faulty). For each pair of adjacent vertices (u,v), u and v can perform the test to each other.

A *test assignment* T for a system G is a collection of tests for every adjacent pairs of vertices. It can be modeled as a directed testing graph $T = (V, L)$ where $(u,v) \in L$ implies that u and v are adjacent in G. Throughout this paper, we assume that each vertex tests the other whenever there is an edge between them and all these tests are gathered in the test assignment. The collection of all test results for a test assignment T is called a *syndrome*. Formally, a syndrome is a function $\sigma : L \to \{0,1\}$.