Applied Mathematics and Computation 218 (2012) 10492–10499

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Dynamic response of a FGPM hollow cylinder under the coupling of multi-fields

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ARTICLE INFO

Keywords:
Functionally graded piezoelectric material (FGPM)
Dynamic response
Hollow cylinder
Electromagnetoelastic
Perturbation of magnetic field vector

ABSTRACT

An analytical study for dynamic electromagnetoelastic responses of a FGPM hollow cylinder, placed in an axial uniform magnetic field, subjected to mechanical loads and electric excitations is presented. The material properties assumed to vary through the radial thickness of the FGPM hollow cylinder according to the same power law function. Based on an interpolation method, by means of finite integral transforms, the numerical results of the dynamic responses of stresses, electric displacement, electric potential and perturbation of magnetic field vector are obtained. The result of investigation may be used as a reference to solve other transient coupled problems of electromagnetoelasticity.

1. Introduction

FGPM is a kind of piezoelectric material with material composition and properties varying continuously along certain directions. FGPM is the composite material intentionally designed so that they posses desirable properties for some specific applications. The advantage of this new kind of materials can improve the reliability of life span of piezoelectric devices. Recently there has been growing interest in materials deliberately fabricated so that their electric, magnetic and mechanical properties vary continuously is space on the macroscopic scale. Zimmerman and Lutz [1] gave the analytical solution for the stresses in FGM cylinders and FGM spheres. Considering the role of thermal and mechanical loads, Tarn [2] investigated an exact solution for functionally graded anisotropic cylinders. Using the infinitesimal theory of elasticity, Naki and Murat [3] obtained the closed-form solutions for stresses and displacements in functionally graded cylindrical and spherical vessels subjected to internal pressure. Based on Lutz and Zimmerman's investigation on FGM symmetric structures, Jabbari et al. [4] investigated the thermal and mechanical stresses in FGM cylinders due to radial symmetric loads. Wu et al. [5] presented an analytical study for piezothermoelastic behavior of a functionally graded piezoelectric cylindrical shell subjected to axisymmetric thermal of mechanical loading. Using the state space formulations, Chen et al. [6] investigated the free vibration of an arbitrarily thick orthotropic piezoelectric hollow cylinder with a functionally graded property along the thickness direction and filled with a non-viscous compressible fluid medium. By virtue of the introduction of a dependant variable and the separation of variables technique, Ding et al. [7] gave an analytical solution of a special non-homogeneous pyroelectric hollow cylinder for piezothermoelastic axisymmetric plane strain dynamic problems. By means of Galekin finite element and Newmark methods, Shakeri et al. [8] presented the analysis of functionally graded hollow cylinders under dynamic load. Shao and Ma [9] carried out thermo-mechanical analysis of FGM hollow cylinders subjected to mechanical loads and linearly increasing boundary temperature. Recently, Dai et al. [10–12] studied the magnetothermoelastic interactions in FGM hollow and solid cylindrical and spherical structures subjected to mechanical loads. To our knowledge, however, investigation

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http://dx.doi.org/10.1016/j.amc.2012.04.013

on the dynamic problem for the FGPM hollow cylinder, placed in an axial uniform magnetic field, under mechanical load and electric excitation has not been found in the literatures.

In this paper, an analytical method is developed for solving the dynamic responses of the FGPM hollow cylinder, placed in an axial uniform magnetic field, subjected to mechanical load and electric excitation. The dynamic responses of stresses, electric potential and perturbation of magnetic field vector in the FGPM hollow cylinder will be calculated. The feature of the solution is related to the propagation of the cylindrical wave.

2. Basic formulations of the problem

A long, FGPM hollow cylinder with internal radius a and external radius b placed in an axial uniform magnetic field $\vec{H}(0,0,H_z)$, let the cylindrical coordinates of any representative point be (r,θ,z). For the axisymmetric plane strain problem, the constitutive relations can be expressed as

$$\sigma_r = c_{11}\frac{\partial u(r,t)}{\partial r} + c_{12}\frac{u(r,t)}{r} + e_{11}\frac{\partial \phi(r,t)}{\partial r}, \tag{1a}$$

$$\sigma_\theta = c_{12}\frac{\partial u(r,t)}{\partial r} + c_{22}\frac{u(r,t)}{r} + e_{12}\frac{\partial \phi(r,t)}{\partial r}, \tag{1b}$$

$$\sigma_z = c_{13}\frac{\partial u(r,t)}{\partial r} + c_{23}\frac{u(r,t)}{r} + e_{13}\frac{\partial \phi(r,t)}{\partial r}, \tag{1c}$$

$$D_r = e_{11}\frac{\partial u(r,t)}{\partial r} + e_{12}\frac{u(r,t)}{r} - g_{11}\frac{\partial \phi(r,t)}{\partial r}, \tag{1d}$$

where $u(r,t)$, $\sigma_i(i=r,\theta,z)$, $\phi(r,t)$ and D_r are the components of displacement, stresses, electric potential and radial electric displacement, respectively. c_{ij}, e_{ij} $(i,j=1,2,3)$ and g_{11} are, respectively, elastic constants, piezoelectric constants and the dielectric constant. All material parameters are assumed to have the same power-law dependence on the wall thickness of the FGPM hollow cylinder, i.e.

$$c_{ij} = c^0_{ij}r^\beta (i=1,2;j=1,2,3), \quad e_{1i} = e^0_{1i}r^\beta (i=1,2,3), \quad g_{11}(r) = g^0_{11}r^\beta, \quad \rho = \rho^0 r^\beta, \quad \mu(r) = \mu^0 r^\beta. \tag{2}$$

Here, subscript zero denotes corresponding value at the outer surface $(r=b)$ of the FGPM hollow cylinder, and β is the non-homogeneous index determined empirically. The range $-2 \leqslant \beta \leqslant 2$ to be used in the present study covers all the values of coordinate exponent encountered in Refs. [3,13,14]. However, these values for β do not necessarily represent a certain material, various β values are used to demonstrate the effect of inhomogeneity.

Omitting displacement electric currents, one obtains the governing electrodynamic Maxwell equations [15,16] for a perfectly conducting, elastic body are given by

$$\vec{J} = \nabla \times \vec{h}, \quad \nabla \times \vec{e} = -\mu(r)\frac{\partial \vec{h}}{\partial t}, \quad div\,\vec{h} = 0, \quad \vec{e} = -\mu(r)\left(\frac{\partial \vec{U}}{\partial t} \times \vec{H}\right), \quad \vec{h} = \nabla \times \left(\frac{\partial \vec{U}}{\partial t} \times \vec{H}\right). \tag{3}$$

where $\vec{J}, \vec{h}, \vec{e}$ and $\vec{U}$ denotes, respectively, electric current density vector, perturbation of magnetic field vector, perturbation of electric field vector and the displacement vector.

Substituting the initial magnetic vector $\vec{H}(0,0,H_z)$ into Eq. (3), yields

$$\vec{U} = (u,0,0), \quad \vec{e} = -\mu(r)\left(0, H_z\frac{\partial u}{\partial t}, 0\right), \quad \vec{h} = (0,0,h_z), \quad \vec{J} = \left(0, -\frac{\partial h_z}{\partial t}, 0\right), \quad h_z = -H_z\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right). \tag{4}$$

By means of taking into account the effect of the Lorentz force f_z, in absence of body forces, the equation of motion for the FGPM hollow cylinder is expressed as

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} + f_z = \rho\frac{\partial^2 u(r,t)}{\partial t^2}, \tag{5}$$

where the Lorentz force f_z can be given as follows:

$$f_z = \mu^0 H_z^2 \frac{\partial}{\partial r}\left(r^\beta \frac{\partial u(r,t)}{\partial r} + r^\beta \frac{u(r,t)}{r}\right). \tag{6}$$

The corresponding mechanical and electrical boundary conditions are expressed as follows, respectively:

$$\sigma_r(i,t) = -p_i(t), \quad \phi(i,t) = \phi_i(t), \quad (i=a,b). \tag{7}$$