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Improved approach to robust stability and $\mathcal{H}_\infty$ performance analysis for systems with an interval time-varying delay

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ABSTRACT

An improved delay- and range-dependent robust stability and $\mathcal{H}_\infty$ performance analysis criterion for systems with an interval time-varying delay is investigated. A novel Lyapunov–Krasovskii functional containing triple-integral terms is established, and an extended free-weighting matrix method, double-integral inequality method is subsequently proposed. Numerical examples validate the effectiveness of the proposed double-integral inequality method and show that our approach is less conservative than existing methods.

1. Introduction

Time-delay phenomena are ubiquitous in practical engineering systems such as biological systems, network systems, chemical engineering systems, and mechanical systems. Such time delays are often the cause of severe performance degradation or even system instabilities [1,2]. Thus, neglecting them may lead to design imperfections and incorrect analysis results. Hence, over the last two decades, a great deal of effort has been made to investigate the stability criteria for systems with time delays. The existing stability criteria can be classified into two categories: delay-independent and delay-dependent. It is well known that the latter criterion is generally less conservative than the former, especially for small delays. Thus, recent research has focused on deriving delay-dependent criteria for the stability analysis of time-delay systems.

Attempts at deriving delay-dependent criteria include the use of the integral-inequality method to bound the cross terms in the derivative of the Lyapunov–Krasovskii functional [3]. The integral-inequality method in [3] was subsequently, applied to the stabilization problem associated with uncertain time-delay systems [4]. Furthermore, a descriptor system approach that involved the integral-inequality method was introduced [5,6]. Recently, with the aim of guaranteeing the asymptotic stability of time-delay systems, free-weighting matrix methods were proposed to reduce the conservatism caused by the use of system transformations and bounding techniques and to enlarge the resultant feasible region for each criterion [7,8]. Even more recently, in order to utilize all the information available, the free-weighting matrix method was extended to the case of a new Lyapunov–Krasovskii functional that contains not only a time-varying delayed state, $x(t-d(t))$, but also a delay-upper-bounded state, $x(t-\bar{d})$, for $0 \leqslant d(t) \leqslant \bar{d}$ [9]. In the present study, we focus on the less-conservative estimation of the upper bound of the derivative of the Lyapunov–Krasovskii functional with triple-integral terms, $\int_{\Omega_1}\int_{\Omega_2}\int_{\Omega_3}(\cdot)d\alpha d\beta d\omega$, which was recently introduced in [10–21]. However, few results using free-weighting matrix methods for the Lyapunov–Krasovskii functional containing triple-integral terms have been reported; therefore, our development of a triple-integral formulism represents a novel approach to solving the problem of interval time-varying delay.

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Motivated by the concerns given above, we introduce herein an extended free-weighting matrix method, double-integral inequality method, for a Lyapunov–Krasovskii functional containing triple-integral terms. Recently, the interval time-varying delay in dynamic systems has received considerable attention in both theoretical and practical studies performed on such systems [22–27]. This time delay, which varies in a fixed interval of time and has a nonzero lower bound, is often observed in networked control systems (NCSs) with a finite but uncertain signal propagation speed. Thus, we derive an improved delay- and range-dependent robust stability and $\mathcal{H}_\infty$ performance analysis criterion for systems with an interval time-varying delay. This criterion is based on the double-integral inequality method and involves the Lyapunov–Krasovskii functional with triple-integral terms. In the derivation, the convex combination approach reported in [9] is adopted for the non-conservative representation of several time-varying terms that result from the derivation of the robust stability and $\mathcal{H}_\infty$ performance analysis criterion.

The rest of this paper is organized as follows. Section 2 considers the stability, robust stability, and $\mathcal{H}_\infty$ performance analysis criteria for systems with a time-varying delay. Section 3 illustrates some simple example applications to verify these criteria. Finally, Section 4 presents some concluding remarks on the approach that has been developed and tested in this study.

Notation. Lebesgue space $\mathcal{L}_{2+} = \mathcal{L}_2[0,\infty)$ consists of square-integrable functions on $[0,\infty)$. Notations $X \geqslant Y$ and $X > Y$ correspond to $X - Y$ being positive semi-definite and positive definite, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry. In addition, for any matrix M, $\mathrm{Sym}\{M\}$ denotes $M + M^T$.

2. Main results

Let us consider the following delayed system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d(t)) + Bw(t) + Gp(t), \\ q(t) &= Ex(t) + E_d x(t-d(t)), \\ z(t) &= Cx(t) + C_d x(t-d(t)) + Dw(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathcal{R}^{n_x}$ and $x(t-d(t)) \in \mathcal{R}^{n_x}$ denote the state and the delayed state, respectively, and $w(t) \in \mathcal{R}^{n_w}$ denotes the disturbance input such that $w(t) \in \mathcal{L}_{2+}$. Here, the state delay $d(t)$ is assumed to be an interval time-varying type of integer: $d_1 \leqslant d(t) \leqslant d_2$, where d_1 and d_2 are known positive integers and $d(t)$ is not differentiable. Here, $p(t) = \Delta(t)q(t)$ and the time-varying nonlinear function $\Delta(t)$ satisfies $\Delta^T(t)\Delta(t) \leqslant \rho^{-2}I$, where $\Delta(t) \in \mathcal{R}^{n_p \times n_q}$. Our aim is to establish a novel delay- and range-dependent robust stability and $\mathcal{H}_\infty$ performance analysis criterion for the system in (1). For this purpose, let us choose a Lyapunov–Krasovskii functional having the following form:

$$V(t) \triangleq V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{2}$$

$$\begin{aligned} V_1(t) &= x^T(t)Px(t), \\ V_2(t) &= \int_{t-d_1}^{t} x^T(\alpha)Q_1x(\alpha)d\alpha + \int_{t-d_2}^{t-d_1} x^T(\alpha)Q_2x(\alpha)d\alpha, \\ V_3(t) &= \int_{-d_1}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha)R_1\dot{x}(\alpha)d\alpha\, d\beta + \int_{-d_2}^{-d_1}\int_{t+\beta}^{t} \dot{x}^T(\alpha)R_2\dot{x}(\alpha)d\alpha\, d\beta, \\ V_4(t) &= \int_{-d_1}^{0}\int_{t+\beta}^{t} x^T(\alpha)S_1x(\alpha)d\alpha\, d\beta + \int_{-d_2}^{-d_1}\int_{t+\beta}^{t} x^T(\alpha)S_2x(\alpha)d\alpha\, d\beta, \\ V_5(t) &= \int_{-d_1}^{0}\int_{\omega}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha)T_1\dot{x}(\alpha)d\alpha\, d\beta\, d\omega + \int_{-d_2}^{-d_1}\int_{\omega}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha)T_2\dot{x}(\alpha)d\alpha\, d\beta\, d\omega, \end{aligned}$$

where P, Q_1, Q_2, R_1, R_2, S_1, S_2, T_1, and T_2 are positive definite matrices. Further, for later convenience, let us define the augmented state $\zeta(t)$ as $\zeta(t) \triangleq \begin{bmatrix} x^T(t) & x^T(t-d_1) & x^T(t-d(t)) & x^T(t-d_2) & \dot{x}^T(t) & w^T(t) & p^T(t) \end{bmatrix}^T \in \mathcal{R}^{n_\zeta}$, $n_\zeta = 5n_x + n_w + n_p$, and then establish block entry matrices e_i $(i = 1, 2, \ldots, 7)$ as, $e_j \in \mathcal{R}^{n_\zeta \times n_x}$ $(j = 1, 2, \ldots, 5)$, $e_6 \in \mathcal{R}^{n_\zeta \times n_w}$, and $e_7 \in \mathcal{R}^{n_\zeta \times n_p}$,

$$\begin{aligned} e_1 &= [I \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T, \quad e_2 = [0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T, \\ e_3 &= [0 \ \ 0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T, \quad e_4 = [0 \ \ 0 \ \ 0 \ \ I \ \ 0 \ \ 0 \ \ 0]^T, \\ e_5 &= [0 \ \ 0 \ \ 0 \ \ 0 \ \ I \ \ 0 \ \ 0]^T, \quad e_6 = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ I \ \ 0]^T, \\ e_7 &= [0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ I]^T, \end{aligned}$$

such that $x(t) = e_1^T\zeta(t)$, $x(t-d_1) = e_2^T\zeta(t)$, $x(t-d(t)) = e_3^T\zeta(t)$, $x(t-d_2) = e_4^T\zeta(t)$, $\dot{x}(t) = e_5^T\zeta(t)$, $w(t) = e_6^T\zeta(t)$, and $p(t) = e_7^T\zeta(t)$. The following lemma, which is based on the integral inequality method reported in [9], introduces a double-integral inequality method, which plays an important role in the derivation of our main results.