Applied Mathematics and Computation 219 (2012) 1345–1353

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Coalition values derived from methods for comparison of coalition influence for games in characteristic function form

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ARTICLE INFO

Keywords:
Game theory
Cooperative games
Influence
Coalitions

ABSTRACT

The authors introduce new values which express coalition influence in the situation of coalition formation. Some examples which show how introduced values work are given. Propositions in this paper provide some properties that proposed values satisfy in the framework of games in characteristic function form.

1. Introduction

Which coalitions will be formed in the situation of group decision making is one of principal problems in group decision making theories. Comparison of coalition influence will be useful to solve this problem [2,4,5]. Kojima and Inohara [3] introduced methods, which are called blockability relations and viability relations, for comparison of coalition influence for games in characteristic function form. These methods are binary relations, which require pairwise comparison of coalitions. In order to know the results of the comparison of the influences of all coalitions, therefore, one needs much computational complexity. So, the authors in this paper propose new values which show coalition influence based on methods introduced by Kojima and Inohara [3] to compare coalition influence easily. Each of the values indicates a coalition's influence by a real number, and the bigger number is, the more influence the coalition has. Two axioms, which are null coalition axiom and symmetry axiom, are introduced, and propositions which show that the proposed values satisfy these axioms are provided.

The structure of this paper is as follows: the framework of games in characteristic function form and definitions of some types of players in the games are presented in the next section. In Section 3, the definitions of the newly proposed coalition values for all coalitions in games are provided, and propositions which validate that proposed coalition values satisfy null coalition axiom and symmetry axiom are given. The last section is devoted for concluding remarks and further research.

2. Frameworks

A framework of games in characteristic function form is introduced in this section. The following definitions in this section are due to [1].

Let $N = \{1, 2, \ldots, n\}$ be a set of n players. Each subset of N is called a *coalition*, and a coalition $S = \{i_1, i_2, \ldots, i_m\}$ is often denoted by $i_1 i_2 \ldots i_m$ for simplicity. A characteristic function $v : 2^N \rightarrow \mathbb{R}$ such that $v(\emptyset) = 0$ assigns a real number to each coalition, where 2^N and $\mathbb{R}$ denote the power set of N and the set of all real numbers, respectively. For each coalition S, $v(S)$ denotes the payoff which the coalition S can obtain through cooperation. A pair (N, v) is said to be a game in characteristic function form with transferable utility, simply called a game in this paper.

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0096-3003/$ - see front matter
http://dx.doi.org/10.1016/j.amc.2012.07.040

The next example is employed throughout this paper to demonstrate how the newly proposed concepts do work.

Example 1. Consider a pair (N, v) such that $N = \{1, 2, 3, 4\}$ and a characteristic function v that $v(\{i\}) = 0$ for all $i \in N$; $v(14) = v(24) = v(34) = 0$; $v(12) = v(13) = v(124) = v(134) = 36$; $v(23) = v(234) = 24$; $v(123) = v(1234) = 42$. Then, (N, v) is a game. □

Some types of players in a game are introduced as follows:

Definition 1 (*Null players [1]*). Consider a game (N, v). For $i \in N$, player i is said to be a *null player*, if and only if $v(S \cup \{i\}) = v(S)$ for all $S \subseteq N \setminus \{i\}$. □

Because a null player brings no contribution toward other coalitions, other coalitions do not have any positive incentive to form coalitions with a null player. In many cases a bigger coalition gains a bigger payoff. A null player, however, does not generate any additional payoff even if he/she joins whatever another coalition.

Example 2. Consider a game (N, v) in Example 1. Then, player 4 is a null player. In fact, $v(4) = 0 = v(\emptyset)$, $v(14) = v(24) = v(34) = 0 = v(1) = v(2) = v(3)$, $v(124) = 36 = v(12)$, $v(134) = 36 = v(13)$, $v(234) = 2 = v(23)$ and $v(1234) = 42 = v(123)$, so that $v(S \cup \{i\}) = v(S)$ for all $S \subseteq N \setminus \{i\}$. □

Definition 2 (*Symmetric players [1]*). Consider a game (N, v). For $i, j \in N$, player i and player j are said to be *symmetric players*, if and only if $v(T \cup \{i\}) = v(T \cup \{j\})$ for all $T \subseteq N \setminus \{i, j\}$. □

Symmetric players i and j have the same contribution when one of them joins a coalition which does contain neither i nor j.

Example 3. Consider a game (N, v) in Example 1. Then, players 2 and 3 are symmetric players. In fact, $v(12) = v(13) = 36$, $v(24) = v(34) = 0$ and $v(124) = v(134) = 36$. □

Next two methods for comparison of coalition influence for games in characteristic function form are introduced. The following definitions are due to Kojima and Inohara [3].

Definition 3 (*Blockability relations for games in characteristic function form [3]*). Consider a game (N, v). For a coalition T, let $B^*(T)$ be $\sum_{U \subseteq N} v(U \setminus T)$. For coalitions S and S', $S \succeq^B S'$ is defined as $B^*(S) \leqslant B^*(S')$. $\succeq^B$ is called the *blockability relation* for (N, v). □

$S \succeq^B S'$ expresses that coalition S can decrease the value of the characteristic function v by deviating from the other coalitions equally to or more than coalition S' can do.

Example 4. Consider a game (N, v) in Example 1. For coalitions 12 and 34, we have

$$B^*(12) = \sum_{U \subseteq N} v(U \setminus 12) = 4 \cdot [v(\emptyset) + v(\{3\}) + v(\{4\}) + v(34)] = 0 \quad \text{and}$$

$$B^*(34) = \sum_{U \subseteq N} v(U \setminus 34) = 4 \cdot [v(\emptyset) + v(\{1\}) + v(\{2\}) + v(12)] = 144.$$

By the definition of blockability relations, we have $12 \succeq^B 34$. □

Definition 4 (*Viability relations for games in characteristic function form [3]*). Consider a game (N, v). For a coalition T, let $V^*(T)$ be $\sum_{U \subseteq N} v(T \setminus U)$. For coalitions S and S', $S \succeq^V S'$ is defined as $V^*(S) \geqslant V^*(S')$. $\succeq^V$ is called the *viability relation* for (N, v). □

$S \succeq^V S'$ expresses that coalition S can defend the value of the characteristic function from the deviation of the other coalitions equally to or more than coalition S' can do.

Example 5. Consider a game (N, v) in Example 1. For coalitions 12 and 34, we have

$$V^*(12) = \sum_{U \subseteq N} v(12 \setminus U) = 4 \cdot [v(\emptyset) + v(\{1\}) + v(\{2\}) + v(12)] = 144 \quad \text{and}$$

$$V^*(34) = \sum_{U \subseteq N} v(34 \setminus U) = 4 \cdot [v(\emptyset) + v(\{3\}) + v(\{4\}) + v(34)] = 0.$$

By the definition of blockability relations, we have $12 \succeq^V 34$. □