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Equilibrium and stability analysis of delayed neural networks under parameter uncertainties

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ABSTRACT

This paper proposes new results for the existence, uniqueness and global asymptotic stability of the equilibrium point for neural networks with multiple time delays under parameter uncertainties. By using Lyapunov stability theorem and applying homeomorphism mapping theorem, new delay-independent stability criteria are obtained. The obtained results are in terms of network parameters of the neural system only and therefore they can be easily checked. We also present some illustrative numerical examples to demonstrate that our result are new and improve corresponding results derived in the previous literature.

1. Introduction

In recent years, the dynamical behavior of neural networks has been extensively studied because of their applications in many different areas such as pattern recognition, optimization, image processing, associative memory design and control systems. When neural networks are employed to solve the problems in the areas of optimization, signal processing neural control systems, the designed neural network must have only one equilibrium point for a particular input vector and this equilibrium point must be globally asymptotically stable to avoid the risk of having multiple equilibrium point. Therefore, the analysis of the existence, uniqueness and global asymptotic stability of the equilibrium point for neural networks is one of the key factors in the design and applications of neural networks. In electronic implementation of neural networks, some time delays occur due to the transmission process, which might turn a stable neural network into an unstable one. On the other hand, parameter uncertainties are unavoidable in neural networks due to the existence of external disturbances and parameter deviations in process of implementation. Therefore, in order to avoid undesired complex and unstable dynamical behaviors, one must study the stability properties of neural networks by taking into account the time delays and parameter uncertainties. In this case, we have to establish the stable equilibrium points that are delay-independent and robust against parameter deviations, leading us to study global robust stability properties of neural networks. In the recent literature, many papers have dealt with obtaining sufficient conditions for existence, uniqueness and global robust asymptotic stability of the equilibrium point for delayed neural networks [1–36]. In this paper, we will present new alternative conditions for global robust stability of delayed neural networks with multiple time delays. We will also give some numerical examples to compare our results with the previous corresponding stability results.

A neural network with multiple time delays is described by the following set of differential equations:

$$\frac{dx_i(t)}{dt} = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij})) + u_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

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where n is the number of the neurons, $x_i(t)$ denotes the state of the neuron i at time t, $f_i(\cdot)$ denote activation functions, a_{ij} and b_{ij} denote the strengths of connectivity between neurons j and i at time t and $t-\tau_{ij}$, respectively; τ_{ij} represents the time delay required in transmitting a signal from the neuron j to the neuron i, u_i is the constant input to the neuron i, c_i is the charging rate for the neuron i.

For the functions f_i, we will assume that there exist some positive constants k_i such that

$$0 \leqslant \frac{f_i(x)-f_i(y)}{x-y} \leqslant k_i, \quad i=1,2,\ldots,n, \quad \forall x,y \in R,\ x \neq y.$$

This class of functions will be denoted by $f \in \mathcal{K}$.

In order to completely characterize the parameter uncertainties, the quantities a_{ij} and b_{ij} and c_i in the neural network model (1) are assumed to satisfy the following parameter ranges:

$$\begin{aligned} C_I &:= \{C = diag(c_i) : 0 < \underline{C} \leqslant C \leqslant \overline{C},\ \text{i.e.,}\ 0 < \underline{c}_i \leqslant c_i \leqslant \bar{c}_i, i=1,2,\ldots,n\}, \\ A_I &:= \{A = (a_{ij}) : \underline{A} \leqslant A \leqslant \overline{A},\ \text{i.e.,}\ \underline{a}_{ij} \leqslant a_{ij} \leqslant \bar{a}_{ij}, i,j=1,2,\ldots,n\}, \\ B_I &:= \{B = (b_{ij}) : \underline{B} \leqslant B \leqslant \overline{B},\ \text{i.e.,}\ \underline{b}_{ij} \leqslant b_{ij} \leqslant \bar{b}_{ij}, i,j=1,2,\ldots,n\}. \end{aligned} \tag{2}$$

Based on the above assumption, the robust asymptotic stability of the equilibrium point for the neural network model (1) is defined as follows:

Definition 1 [20]. The neural network model (1) with the parameter ranges defined by (2) is globally asymptotically robust stable if the unique equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ of the neural network (1) is globally asymptotically stable for all $C \in C_I$, $A \in A_I$ and $B \in B_I$.

Throughout this paper, we will use the following notations: Let $v = (v_1, v_2, \ldots, v_n)^T$be a vector of dimension n and $Q = (q_{ij})$ be a real $n \times n$ matrix. Then, $|v|$ will denote $|v| = (|v_1|, |v_2|, \ldots, |v_n|)^T$ and $|Q|$ will denote $|Q| = (|q_{ij}|)_{n\times n}$. The following norms will also be used:

$$\|v\|_1 = \sum_{i=1}^n |v_i|, \quad \|v\|_2 = \left\{\sum_{i=1}^n |v_i|^2\right\}^{1/2}, \quad \|v\|_\infty = \max_{1\leqslant i\leqslant n} |v_i|,$$

$$\|Q\|_1 = \max_{1\leqslant i\leqslant n} \sum_{j=1}^n |q_{ji}|, \quad \|Q\|_2 = [\lambda_{\max}(Q^TQ)]^{1/2}, \quad \|Q\|_\infty = \max_{1\leqslant i\leqslant n} \sum_{j=1}^n |q_{ij}|.$$

In what follows, we state the homeomorphism mapping theorem which will be used in the proof of the existence and uniqueness of the equilibrium point:

Lemma 1 [1]. *If $H(x) \in C^0$ satisfies the conditions $H(x) \neq H(y)$ for all $x \neq y$ and $\|H(x)\| \to \infty$ as $\|x\| \to \infty$ then, $H(x)$ is homeomorphism of R^n.*

The following definitions and lemmas which will help to make a precise comparison between our stability conditions and previously reported results in the literature are restated in the following:

Definition 2 [37]. A real $n \times n$ matrix $Q = (q_{ij})$ is said to be a nonsingular M-matrix if $q_{ii} > 0$ and $q_{ij} \leqslant 0, i,j = 1,\ldots,n,\ i \neq j$; and the real part of every eigenvalue of Q is positive.

Lemma 2 [37]. *Let $Q = (q_{ij})$ be a $n \times n$ matrix with positive diagonal entries and non-positive off-diagonal entries. Then, Q is a nonsingular M-matrix if and only if there exists a positive vector $\Psi > 0$ such that $Q\Psi > 0$.*

In order to introduce the next lemma, we will first observe that if A is real matrix with $A \in A_I := \{A = (a_{ij}) : \underline{A} \leqslant A \leqslant \overline{A},\ \text{i.e.,}\ \underline{a}_{ij} \leqslant a_{ij} \leqslant \bar{a}_{ij}, i,j=1,2,\ldots,n\}$. Then, a_{ij} can be expressed as follows:

$$a_{ij} = \frac{1}{2}(\bar{a}_{ij} + \underline{a}_{ij}) + \frac{1}{2}\sigma_{ij}(\bar{a}_{ij} - \underline{a}_{ij}), \quad -1 \leqslant \sigma_{ij} \leqslant 1, \quad i,j = 1,2,\ldots,n.$$

Define $A^* = \frac{1}{2}(\overline{A} + \underline{A}), A_* = \frac{1}{2}(\overline{A} - \underline{A})$, and $\widetilde{A} = (\tilde{a}_{ij})_{n\times n}$ with $\tilde{a}_{ij} = \frac{1}{2}\sigma_{ij}(\bar{a}_{ij} - \underline{a}_{ij})$, where $-1 \leqslant \sigma_{ij} \leqslant 1, \quad i,j = 1,2,\ldots,n$. In this case, A can be expressed as follows:

$$A = \frac{1}{2}(\overline{A} + \underline{A}) + \widetilde{A} = A^* + \widetilde{A}.$$