



Positive periodic solutions for impulsive Gause-type predator–prey systems

Xiaoquan Ding^{a,*}, Bentang Su^b, Jianmin Hao^b

^a School of Mathematics and Statistics, Henan University of Science and Technology, Luoyang, Henan 471003, PR China

^b Department of Mathematics and Information Science, Shandong Agricultural University, Tai'an, Shandong 271018, PR China

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ABSTRACT

This paper is devoted to impulsive periodic Gause-type predator–prey systems with monotonic or non-monotonic numerical responses. With the help of a continuation theorem based on coincidence degree theory, we establish easily verifiable criteria for the existence of positive periodic solutions. As corollaries, some applications are listed. In particular, our results improve and generalize some known ones.

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1. Introduction

Existence of positive periodic solutions for population dynamical systems has been intensively investigated in the literature including [1–10], and references therein.

In [3], Ding and Jiang studied the following periodic Gause-type predator–prey system:

$$\begin{cases} x'(t) = x(t)f(t, x(t - \tau(t))) - g(t, x(t))y(t - \sigma_1(t)), \\ y'(t) = y(t)[-d(t) + h(t, x(t - \sigma_2(t)))] \end{cases} \quad (1.1)$$

where x and y are the prey and the predator population size, respectively. The function $f(t, v)$ is the growth rate of the prey in the absence of the predator. The function $d(t)$ is the death rate of the predator. The function $g(t, v)$, called functional response of predator to prey, describes the change in the rate of exploitation of prey by a predator as a result of a change in the prey density. The function $h(t, v)$, called numerical response of predator to prey, describes the change in reproduction rate with changing prey density. Using the method of coincidence degree, the authors established sufficient conditions for the existence of positive periodic solutions.

As we know, in population dynamics, many evolutionary processes experience short-time rapid change after undergoing relatively long smooth variation. Examples include annual harvesting and stocking of species as well as annual immigration. Incorporating these phenomena gives us impulsive differential equations. For the theory of impulsive differential equations, we refer the reader to [11,12]. In the past few years, there have been a number of studies which applied impulsive differential equations to biological problems (see, e.g. [13–18]).

In the present paper, incorporating impulses into system (1.1), we investigate the following impulsive periodic Gause-type predator–prey system:

$$\begin{cases} \left. \begin{aligned} x'(t) &= x(t)f(t, x(t - \tau(t))) - g(t, x(t))y(t - \sigma_1(t)), \\ y'(t) &= y(t)[-d(t) + h(t, x(t - \sigma_2(t)))] \end{aligned} \right\} t \neq t_k, \\ \left. \begin{aligned} \Delta x(t) &= a_k x(t), \\ \Delta y(t) &= b_k y(t), \end{aligned} \right\} t = t_k, \end{cases} \quad (1.2)$$

* Corresponding author.

E-mail address: xqding@mail.haust.edu.cn (X. Ding).

where $\Delta x(t) = x(t^+) - x(t)$, $\Delta y(t) = y(t^+) - y(t)$, $0 < t_1 < t_2 < \dots < t_k < \dots$ are fixed impulsive points, and $\lim_{k \rightarrow \infty} t_k = +\infty$.

For the sake of generality and convenience, we always make the following fundamental assumptions for system (1.2):

- (H₁) $\tau(t)$, $\sigma_1(t)$, $\sigma_2(t)$ and $d(t)$ are periodic continuous functions with a common period $\omega > 0$.
 (H₂) a_k and b_k are constants with $1 + a_k > 0$, $1 + b_k > 0$, and there exists an integer $q > 0$ such that $a_{k+q} = a_k$, $b_{k+q} = b_k$, and $t_{k+q} = t_k + \omega$ for $k \in \mathbb{Z}^+$. Without loss of generality, we assume that $[0, \omega] \cap \{t_k\}_{k=1}^{\infty} = \{t_1, t_2, \dots, t_q\}$.
 (H₃) $f(t, v)$, $g(t, v)$ and $h(t, v)$ are continuous functions and ω -periodic in t , $(\partial f / \partial v)(t, v)$, $(\partial g / \partial v)(t, v)$ and $(\partial h / \partial v)(t, v)$ are also continuous functions.
 (H₄) There exists a continuous ω -periodic function $r(t)$ such that $f(t, v) \leq r(t)$ for $t \in \mathbb{R}$, $v > 0$.
 (H₅) There exists a positive constant c_0 such that $0 < g(t, v) \leq c_0 v$ for $t \in \mathbb{R}$, $v > 0$.
 (H₆) $h(t, 0) = 0$, $h(t, v) > 0$ for $t \in \mathbb{R}$, $v > 0$, and

$$0 < \int_0^\omega d(t) dt - \sum_{k=1}^q \ln(1 + b_k) < \sup_{v \geq 0} \int_0^\omega h(t, v) dt.$$

Readers familiar with predator–prey models may notice that the above assumptions are reasonable for population models [19]. The assumption of periodicity of the parameters is a way of incorporating the periodicity of the environment. From the viewpoint of mathematical biology, we only consider positive solutions of system (1.2). Therefore, it is a natural constraint that $1 + a_k$ and $1 + b_k$ are all positive. Under the above assumptions, system (1.2) covers many models appeared in the literature. For instance, $g(t, v)$ can be taken as the logistic growth [1,4,7,18], the Gilpin growth $a - bv^\theta$ [20], the Smith growth $(a - bv)/(D + v)$ [21], and the Allee effect $a + bv - cv^2$ [22]. $h(t, v)$ can be taken as functional responses of the Lotka–Volterra type mv [19], the Holling type $mx^n/(A + x^n)$ [7,23], the Ivlev type $m(1 - e^{-Ax})$ [24], and the Monod–Haldane type $mx/(A + x^2)$ [25]. $h(t, v)$ can be taken as $c(t)g(t, v)$, where $c(t)$ is a continuous ω -periodic function.

We note that Wang et al. [18] have considered a special case of system (1.2), and established sufficient conditions for the existence of positive periodic solutions. So far as we know, there is no published paper concerned system (1.2).

The main purpose of this paper is, by using the coincidence degree theory developed by Gaines and Mawhin [26], to derive a set of easily verifiable sufficient conditions for the existence of one or multiple positive periodic solutions of system (1.2). Furthermore, we will see that our results for the above system can be easily extended to the ones with a distributed or state-dependent delay. As corollaries, some applications are listed. In particular, our results improve and generalize theorems in [1–4,7,18].

2. Preliminaries

In this section, we recall the continuation theorem, the compactness criterion and an auxiliary inequality, which play an important role in the proof of our main results.

Define

$$PC_\omega = \left\{ \varphi : \mathbb{R}^+ \rightarrow \mathbb{R} \mid \begin{array}{l} \text{(i) } \varphi \text{ is continuous at } t \neq t_k, \text{ left continuous at } t = t_k, \text{ and } \lim_{t \rightarrow t_k+0} \varphi(t) \text{ exists, } k \in \mathbb{Z}^+, \\ \text{(ii) } \varphi(t + \omega) = \varphi(t) \end{array} \right\},$$

$$PC_\omega^1 = \left\{ \varphi \in PC_\omega \mid \varphi \text{ is continuous differential at } t \neq t_k, \lim_{t \rightarrow t_k-0} \varphi'(t) \text{ and } \lim_{t \rightarrow t_k+0} \varphi'(t) \text{ exist, } k \in \mathbb{Z}^+ \right\}.$$

We note that if $\varphi \in PC_\omega^1$ then $\varphi' \in PC_\omega$.

The set $\mathcal{F} \subset PC_\omega$ is said to be quasi-equicontinuous if for any $\varepsilon > 0$ there exists a $\delta > 0$ such that if $\varphi \in \mathcal{F}$, $k \in \mathbb{Z}^+$, t' , $t'' \in (t_{k-1}, t_k] \cap [0, \omega)$, and $|t' - t''| < \delta$, then $|\varphi(t') - \varphi(t'')| < \varepsilon$.

Let X, Y be real Banach spaces, $L : \text{Dom} L \subset X \rightarrow Y$ be a linear mapping, and $N : X \rightarrow Y$ be a continuous mapping.

The mapping L is said to be a Fredholm mapping of index zero, if $\dim \text{Ker} L = \text{codim} \text{Im} L < +\infty$ and $\text{Im} L$ is closed in Y .

If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \rightarrow X$ and $Q : Y \rightarrow Y$, such that $\text{Im} P = \text{Ker} L$, $\text{Ker} Q = \text{Im} L = \text{Im}(I - Q)$. It follows that the restriction L_P of L to $\text{Dom} L \cap \text{Ker} P : (I - P)X \rightarrow \text{Im} L$ is invertible. Denote the inverse of L_P by K_P .

The mapping N is said to be L -compact on $\bar{\Omega}$, if Ω is an open bounded subset of X , $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \rightarrow X$ is compact.

Since $\text{Im} Q$ is isomorphic to $\text{Ker} L$, there exists an isomorphism $J : \text{Im} Q \rightarrow \text{Ker} L$.

Lemma 2.1 (Compactness criterion [11, p. 24]). *The set $\mathcal{F} \subset PC_\omega$ is relatively compact if and only if*

- \mathcal{F} is bounded, that is, $\|\varphi\|_{PC_\omega} = \sup_{t \in [0, \omega]} |\varphi(t)| \leq M$ for each $\varphi \in \mathcal{F}$ and some $M > 0$;
- \mathcal{F} is quasi-equicontinuous.

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