Applied Mathematics and Computation 218 (2012) 6851–6859

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Hopfield neural networks in large-scale linear optimization problems

Marta I. Velazco Fontova [a], Aurelio R.L. Oliveira [b], Christiano Lyra [c,*]

[a] Faculty of Campo Limpo Paulista-FACCAMP, Rua Guatemala 167, Bairro Jd. América, 13231-230 Campo Limpo Paulista, SP, Brazil
[b] Institute of Mathematics, Statistics and Scientific Computing (IMECC), University of Campinas (UNICAMP), Praça Sérgio Buarque de Holanda 651, CP 6065, 13083-859 Campinas, SP, Brazil
[c] Department of Systems Engineering (DENSIS), School of Electrical and Computer Engineering (FEEC), University of Campinas (UNICAMP), Av. Albert Einstein 400, CP 6101, 13083-852 Campinas, SP, Brazil

ARTICLE INFO

Keywords:
Hopfield networks
Optimization
Interior point methods
Affine scaling methods
Linear programming
Neural networks

ABSTRACT

Hopfield neural networks and affine scaling interior point methods are combined in a hybrid approach for solving linear optimization problems. The Hopfield networks perform the early stages of the optimization procedures, providing enhanced feasible starting points for both primal and dual affine scaling interior point methods, thus facilitating the steps towards optimality. The hybrid approach is applied to a set of real world linear programming problems. The results show the potential of the integrated approach, indicating that the combination of neural networks and affine scaling interior point methods can be a good alternative to obtain solutions for large-scale optimization problems.

1. Introduction

Large-scale linear optimization problems are computationally time-consuming, and various interior point methods have been adopted for their solution [1,5,13,15,22,25]; primal–dual path following versions [25] tend to be preferred. However, in certain situations, affine-scaling versions may be the most efficient approach [1,16,22].

Other investigations for the solution of linear optimization problems have explored the possibilities of Hopfield neural networks [21,26,27]. This possibility was pioneered by Tank and Hopfield [21]. Xia and Wang [26,27] later proposed a global convergent neural network for solving these problems. Other approaches have recently been used for solving combinatorial [20] and nonlinear [6] problems. However, the problems explored with all approaches have been relatively limited in size.

The initial solution for interior point methods must be interior and, for some methods, feasible. Advanced initial points are exploited by many authors because a warm start can speed up the optimization process. Yildirim and Wright [28], for instance, computed an advanced initial point by solving a perturbed problem. In this case, the point is very poorly centered and the method needs more steps to get away from the boundary [17]. Other approach was presented by Gondzio [11] with cutting plane methods; however, the problem size increases, what precludes its use for large-scale problems.

A new trend in the past few years consists in the use of simple linear programming methods in order to give a warm starting point for interior point methods, thus reducing the total number of iterations [9]. The von Neumann's algorithm, proposed to Dantzig [4], is one of the first methods used in such applications since its iteration is inexpensive [7]. The idea presented in [9] to develop the optimal pair adjustment algorithm was generalized for p coordinates in [19], where p is bounded by the problem dimension. For each p there is a different algorithm and, consequently, a family of algorithms is obtained.

* Corresponding author.
E-mail addresses: marta.velazco@gmail.com (M.I. Velazco Fontova), aurelio@ime.unicamp.br (A.R.L. Oliveira), chrlyra@densis.fee.unicamp.br (C. Lyra).

0096-3003/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2011.12.059

It has recently been pointed out that a combination of interior point methods and neural networks [12] can provide benefits in running times [23], since the initial use of Hopfield neural networks gives points closer to an optimal solution. The primal–dual interior point method obtains benefits from these more adequate starting points, thus paving the path towards optimality. Moreover, the algebraic systems used in Hopfield neural networks and interior point methods are similar, allowing the simple exchange of information and solution techniques.

This paper investigates new hybrid approaches combining Hopfield neural networks and the affine scaling family of interior point optimization methods. Initially, neural networks are used to compute feasible interior starting points for both primal and dual affine scaling methods. Another approach investigates the use of Hopfield networks striving to improve starting points, advancing along the path to optimality. Both hybrid approaches are applied to a set of real-life linear programming problems in the *Netlib*[1] collection [8].

The remainder of the paper is organized as follows. Section 2 presents the primal and dual affine scaling methods for linear optimization. The Hopfield approach for the solution of optimization problems is reviewed in Section 3. Section 4 discusses improvements in the Hopfield approach [18] for linear programming. Hybridization of Hopfield neural networks and the affine scaling family of interior point optimization methods is described in Section 5. Case studies are presented in Section 6. Conclusions are presented in the final section.

2. Affine scaling interior point methods

Dikin [5] pioneered the idea of interior point methods to replace the *Simplex* method in the solution of linear optimization problems. He proposed the "primal affine scaling method". Karmarkar [13] later formalized the convergence properties of an interior point approach and presented a method with better convergence properties than the simplex methods. More recently, new methods exploiting the interior point approach have been introduced (e.g. [25]).

The linear optimization problem can be stated in primal form as follows [14]

$$\begin{aligned} \min \quad & c^T \cdot x \\ \text{subject to} \quad & A \cdot x = b \\ & x \geqslant 0 \end{aligned} \tag{1}$$

where $A \in \Re^{m\times n}$.

Associated with the problem (1), is the dual problem:

$$\begin{aligned} \max \quad & b^T \cdot y \\ \text{subject} \rightarrow \quad & A^T \cdot y + z = c \\ & z \geqslant 0 \end{aligned} \tag{2}$$

The optimality conditions for these problems, both primal and dual, are given by primal and dual feasibility, together with the *complementarity conditions*,

$$X \cdot Z \cdot e = 0 \tag{3}$$

where $X = diag(x)$ and $Z = diag(z)$ are diagonal matrixes with the diagonals formed by vectors x and z, respectively. Vector e is a column vector consisting only of ones. In these problems, a point is said to be interior if there are no binding variables (in other words, if $x > 0$ for the primal problem and $z > 0$ for the dual problem). Such primal and dual problems can be solved using primal and dual affine-scaling interior point methods, respectively.

2.1. Primal affine scaling method

The primal affine scaling method can be briefly described by the following sequence of steps [22].

Given a feasible interior point for problem (1), $x^\circ > 0$, do, for $k = 0, 1, 2, \ldots,$

1. Compute an estimate for the two dual variables y^k and z^k;
2. Compute the search direction, $\Delta x^k = -(X^k)^2 z^k$;
3. Choose an appropriate step length to maintain the next point interior, α^k;
4. Compute the new interior point, $x^{k+1} = x^k + \alpha^k \Delta x^k$; continue until convergence is achieved.

2.2. Dual affine scaling method

The dual affine scaling method can be summarized as follows [1].

Given a feasible interior point for the problem (2), y°, z° ($z^\circ > 0$), do, for $k = 0, 1, 2, \ldots,$

[1] http://www.netlib.org.