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Convergence of numerical solutions to stochastic age-structured system of three species ☆

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ABSTRACT

In general, most of stochastic age-structured system of three species do not have explicit solutions, thus numerical approximation schemes are invaluable tools for exploring their properties. The aim of this paper is to investigate the convergence of numerical approximation solution to the true solution for stochastic age-structured system of three species.

1. Introduction

Stochastic differential equations have been found many applications in such as economics, biology, finance and other sciences. For example, Cadenillas [1] gave a stochastic maximum principle for systems with jumps, with applications to finance, and analysis and control of age-dependent population dynamics by Lowen [2]. One of the most important and interesting problems in the analysis of stochastic differential equations is their numerical solution. For example, Platen [3] gave an introduction to numerical methods for stochastic differential equations, and Mao [4] gave stochastic differential equations and applications.

Recently, the random behavior of the birth–death process is incorporated into the continuous-time age-structured population equations to obtain a system of stochastic differential equations that model age-structured dynamics. However, an application of the stochastic age-structured model is to study how age-structured influences estimated persistence time of a population where extinction is influenced by random fluctuations in the birth–death process. In this paper, we shall discuss the convergence of stochastic partial differential equations. That is, we introduce the following convergence of stochastic age-structured system of food chain:

$$\frac{\partial P_1}{\partial t}+\frac{\partial P_1}{\partial r}=-\mu_1(r,t,x)P_1-\lambda_1(r,t,x)P_2P_1+\mu_1(r,t,x)P_1\frac{\partial W}{\partial t},\quad \text{in}\quad Q_A=(0,A)\times Q, \tag{1.1}$$

$$\frac{\partial P_2}{\partial t}+\frac{\partial P_2}{\partial r}=-\mu_2(r,t,x)P_2+\lambda_2(r,t,x)P_1P_2-\lambda_3(r,t,x)P_3P_2+\mu_2(r,t,x)P_2\frac{\partial W}{\partial t},\quad \text{in}\quad Q_A=(0,A)\times Q, \tag{1.2}$$

$$\frac{\partial P_3}{\partial t}+\frac{\partial P_3}{\partial r}=-\mu_3(r,t,x)P_3+\lambda_4(r,t,x)P_2P_3+\mu_3(r,t,x)P_3\frac{\partial W}{\partial t},\quad \text{in}\quad Q_A=(0,A)\times Q, \tag{1.3}$$

$$P_i(0,t,x)=\int_0^A\beta_i(r,t,x)P_i(r,t,x)dr,\quad \text{in}\quad (0,T)\times\Gamma, \tag{1.4}$$

$$P_i(r,0,x)=P_{i0}(r,x),\quad \text{in}\quad Q'_A=(0,A)\times\Gamma, \tag{1.5}$$

$$P_i(r,t,x)=0,\quad \text{on}\quad \Sigma_A=(0,A)\times(0,T)\times\partial\Gamma, \tag{1.6}$$

$$P_i(r,t,x)=0,\quad \forall r\geqslant A, \tag{1.7}$$

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where $Q = (0,T) \times \Gamma$, the final time $T > 0$, $A > 0$ is the maximal age of the three species, the age cannot reach the maximal with all of the species, so the three population's quantities are zero while the age is A, that is $P_i(A,t,x) = 0$. $\Gamma \subset R^N (N \in \{1,2,3\})$ is an open and bounded habitat where the three species can be free to move, $\partial\Gamma$ is the boundary of Γ, $t \in (0,T)$, $r \in (0,A)$, $x \in \Gamma \cdot P_i(r,t,x)$ denotes of the species density of age r at time t and in the location x, $\beta_i(r,t,x)$ denotes the fertility rate of females of age r at time t and in spatial position x, $\mu_i(r,t,x)$ denotes the mortality rate of age r at time t and in the location x, $\lambda_k(r,t,x)$ is the interaction coefficients of age r at time t and in the location $x (k = 1,2,3,4)$. $\mu_i(r,t,x)P_i\frac{\partial W}{\partial t}$ denotes effects of external environment for population system, such as emigration and earthquake and so on. The effects of external environment has the deterministic and random parts which depend on r, t, x and P_i (for the sake of convenience, throughout this paper, we suppose that $i = 1,2,3$).

The effects of the stochastic environmental noise considerations lead to stochastic age-structured population systems, which are more realistic. For example, Zhang et al. [5] showed the existence, uniqueness and exponential stability for stochastic age-dependent population, and numerical analysis for stochastic age-dependent population equations has been studied by Zhang and Han [6]. However, their results are not concerned with interacting species system of food chain.

The food chain system has received many attentions from several authors. For example, Luo et al. [7] gave optimal birth control for predator–prey system of three species with age-structure, and chaotic behavior of a three-species beddington-type system with impulsive perturbations by Wang et al. [8]. However, their studies are not concerned with stochastic.

In general, stochastic age-structured population Eqs. (1.1)–(1.3) rarely have explicit solutions. Thus, numerical approximation schemes are invaluable tools for exploring its properties. In this paper, we will develop a numerical approximation method for stochastic age-structured system equations of three species of the type described by Eqs. (1.1)–(1.7). The numerical solutions are defined by implicit equations containing partial derivative.

The remainder of this paper is organized as follows: In Section 2, we begin with some preliminary results which are essential for our analysis, and define numerical approximate solutions to stochastic age-structured equations of three species. In Section 3, we shall prove that the numerical solutions converge to the exact solutions and provide the order of convergence.

2. Preliminaries and approximation

Consider stochastic age-structured system equations of three species (1.1)–(1.7). Integrating on $[0,A]$ to Eqs. (1.1)–(1.3) with respect to r, then we obtain the following system:

$$\frac{\partial y_1}{\partial t} - \beta_1(t,x)y_1 + \mu_1(t,x)y_1 + \lambda_1(t,x)y_2y_1 = \mu_1(t,x)y_1\frac{dW}{dt}, \quad \text{in} \quad Q = (0,T)\times\Gamma, \tag{2.1}$$
$$\frac{\partial y_2}{\partial t} - \beta_2(t,x)y_2 + \mu_2(t,x)y_2 - \lambda_2(t,x)y_1y_2 + \lambda_3(t,x)y_3y_2 = \mu_2(t,x)y_2\frac{dW}{dt}, \quad \text{in} \quad Q = (0,T)\times\Gamma, \tag{2.2}$$
$$\frac{\partial y_3}{\partial t} - \beta_3(t,x)y_3 + \mu_3(t,x)y_3 - \lambda_4(t,x)y_2y_3 = \mu_3(t,x)y_3\frac{dW}{dt}, \quad \text{in} \quad Q = (0,T)\times\Gamma, \tag{2.3}$$
$$y_i(t,x) = \int_0^A P_i(r,t,x)dr, \quad \text{in} \quad Q = (0,T)\times\Gamma,$$
$$y_i(0,x) = y_{i0}(x), \quad \text{in} \quad \Gamma,$$
$$y_i(t,x) = 0, \quad \text{on} \quad \Sigma = (0,T)\times\partial\Gamma,$$

where

$$\beta_i(t,x) \equiv \left(\int_0^A \beta_i(r,t,x)P_i(r,t,x)dr\right)\left(\int_0^A P_i(r,t,x)dr\right)^{-1},$$

$\int_0^A P_i(r,t,x)dr$ is the total population, and the birth process is described by the nonlocal boundary conditions $\int_0^A \beta_i(r,t,x)P_i(r,t,x)dr$, clearly, $\beta_i(t,x)$ denotes the fertility rate of total populations at time t and in the location x.

$$\mu_i(t,x) \equiv \left(\int_0^A \mu_i(r,t,x)P_i(r,t,x)dr\right)\left(\int_0^A P_i(r,t,x)dr\right)^{-1},$$

$\mu_i(t,x)$ denotes the mortality rate at time t and in the location x.

$$\lambda_1(t,x) \equiv \left(\int_0^A \lambda_1(r,t,x)P_2P_1dr\right)\left(\int_0^A P_2(r,t,x)dr\right)^{-1}\left(\int_0^A P_1(r,t,x)dr\right)^{-1},$$
$$\lambda_2(t,x) \equiv \left(\int_0^A \lambda_2(r,t,x)P_1P_2dr\right)\left(\int_0^A P_2(r,t,x)dr\right)^{-1}\left(\int_0^A P_1(r,t,x)dr\right)^{-1},$$
$$\lambda_3(t,x) \equiv \left(\int_0^A \lambda_3(r,t,x)P_3P_2dr\right)\left(\int_0^A P_3(r,t,x)dr\right)^{-1}\left(\int_0^A P_2(r,t,x)dr\right)^{-1},$$
$$\lambda_4(t,x) \equiv \left(\int_0^A \lambda_4(r,t,x)P_2P_3dr\right)\left(\int_0^A P_2(r,t,x)dr\right)^{-1}\left(\int_0^A P_3(r,t,x)dr\right)^{-1}.$$