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Decision risk analysis for an interval TOPSIS method

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ABSTRACT

TOPSIS is a multi-attribute decision making (MADM) technique for ranking and selection of a number of externally determined alternatives through distance measures. When the collected data for each criterion is interval and the risk attitude for a decision maker is unknown, we present a new TOPSIS method for normalizing the collected data and ranking the alternatives. The results show that the decision maker with different risk attitude ranks the different alternatives.

1. Introduction

Technique for order performance by similarity to ideal solution (TOPSIS), initially presented by Hwang and Yoon [4], and Lai et al. [7], is a multi-attribute or multi-criteria decision making (MADM/MCDM) [1,4,20] to identify solutions from a finite set of alternatives based on minimum distance from an ideal point and maximum distance from a negative ideal point. Shih [10] exploited incremental analysis to overcome the drawbacks of ratio scales in various MCDM techniques. Shih et al. [11] proposed that the advantages of TOPSIS are represents the rationale of human choice; accounts for both the best and worst alternatives; the performance measures of all alternatives on attributes. In recent years, TOPSIS has been successfully applied to the areas of transportation [14], product design [8], and supply chain management [12]. However, uncertain data may be not precisely determined since human judgments are often vague under insufficient information. Therefore, fuzzy values or interval values are usually collected in measuring of the relative importance of criteria and the performance of each alternative on TOPSIS model. For example, Yang and Hung [18] used fuzzy TOPSIS to solve a plant layout design problem, Jahanshaloo et al. [5,6] presented the TOPSIS model for interval data and proposed another method for ranking the score of each alternative. However, there are two major drawbacks for TOPSIS method. The first drawback is the operation of normalized decision matrix in which the normalized scale for each criterion is usually derived a narrow gap among the performed measures. That is, a narrow gap in the TOPSIS method is not good for ranking and cannot reflect the true dominance of alternatives. Another drawback is that we never considered the risk assessment for a decision maker in the TOPSIS method. According to risk propensity, it has been commonly observed that decision makers differ in that willingness to overestimate the probability of a gain or a loss, the risk attitudes for a decision maker is usually categorized as risk-seeking, risk-neutral, and risk-averse. Without considering risk propensity, the subjective propensity associated with different decision maker preference cannot be determined. In order to cope with these two drawbacks, we summarize the aims of our study as follows:

(1) A new normalized method is proposed as that we can derive a wider gap among the performed measures.
(2) The effect of the risk attitude is considered in the TOPSIS method.

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The paper is organized as follows. Section 2, the TOPSIS method is introduced. In Section 3, in the proposed TOPSIS method, a new normalized method for each criterion with interval value and the risk perception for decision makers are considered. Also, an example for the proposed method is illustrated in Section 4. Finally, we state our conclusions in Section 5.

2. The TOPSIS model

TOPSIS method is a technique for order preference by similarity to ideal solution that maximizes the benefit criteria/attributes and minimizes the cost criteria/attributes, whereas the negative ideal solution maximizes the cost criteria/attributes and minimizes the benefit criteria/attributes. The best alternative is the one, which is closest to the ideal solution and farthest from the negative ideal solution. Suppose a MCDM problem has n alternatives, $A_1, A_2, \ldots, A_n$, and m decision criteria/attributes, $C_1, C_2, \ldots, C_m$. Each alternative is evaluated with respect to the m criteria/attributes. Each value assigned to each alternative with respect to each criterion form a decision matrix denoted by $\mathbf{X} = (x_{ij})_{n\times m}$ as below:

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2m} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{im} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nj} & \cdots & x_{nm} \end{bmatrix}. \tag{1}$$

Let $W = (w_1, w_2, \ldots, w_m)$ be the relative weight vector about the criteria, satisfying $\sum_{j=1}^{m} w_j = 1$. Then the procedure of TOPSIS can be expressed in a series of steps:

Step 1. Calculate the normalized decision matrix. Some normalized methods for TOPSIS are summarized by Shih et al. [11]. For simplify, a vector normalization method is introduced whose normalized value n_{ij} is calculated as:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{n} x_{kj}^2}}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m. \tag{2}$$

Step 2. Calculate the weighted normalized decision matrix $\mathbf{V} = (v_{ij})_{n\times m}$:

$$v_{ij} = w_j n_{ij}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m, \tag{3}$$

where w_j is the relative weight of the jth criterion/attribute, and $\sum_{j=1}^{m} w_j = 1$.

Step 3. Determine the positive ideal A^+ and negative ideal solution A^- as below:

$$A^+ = \{v_1^+, v_2^+, \ldots, v_m^+\} = \left\{ \left(\max_i v_{ij} | j \in \Omega_b \right), \left(\min_i v_{ij} | j \in \Omega_c \right) \right\}, \tag{4}$$

$$A^- = \{v_1^-, v_2^-, \ldots, v_m^-\} = \left\{ \left(\min_i v_{ij} | j \in \Omega_b \right), \left(\max_i v_{ij} | j \in \Omega_c \right) \right\}, \tag{5}$$

where Ω_b is associated with benefit criteria, and Ω_c is associated with cost criteria.

Step 4. Calculate the separation measures, using the m-dimensional Euclidean distance. The separation of each alternative from the ideal solution (A^+) and the negative ideal solution (A^-) are given as below, respectively:

$$D_i^+ = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - v_j^+ \right)^2}, \quad i = 1, 2, \ldots, n, \tag{6}$$

$$D_i^- = \sqrt{\sum_{j=1}^{m} \left(v_{ij} - v_j^- \right)^2}, \quad i = 1, 2, \ldots, n. \tag{7}$$

Step 5. Calculate the relative closeness of each alternative to the ideal solution. The relative closeness of the alternative A_i with respect to A^+ is defined as:

$$RC_i = \frac{D_i^+}{D_i^+ + D_i^-}, \quad i = 1, 2, \ldots, n. \tag{8}$$

Step 6. Rank the alternatives according to the relative closeness to the ideal solution. The smaller the value RC_i, the less distance the alternative A_i to the ideal solution. The best alternative is the one with the greatest relative closeness to the ideal solution.