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Positive periodic solutions for neutral delay ratio-dependent predator–prey model with Holling type III functional response ☆

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ABSTRACT

By using a continuation theorem based on coincidence degree theory, some new and interesting sufficient conditions are obtained for the existence of positive periodic solutions for neutral delay ratio-dependent predator–prey model with Holling type III functional response

$$\begin{cases} x'(t) = x(t)[r(t) - a(t)x(t-\sigma_1) - \rho x'(t-\sigma_2)] - \frac{c(t)x^2(t)}{m^2y^2(t)+x^2(t)}y(t), \\ y'(t) = y(t)\left[-d(t) + \frac{h(t)x^2(t-\tau(t))}{m^2y^2(t-\tau(t))+x^2(t-\tau(t))}\right]. \end{cases}$$

An example is represented to illustrate the feasibility of our main results.

1. Introduction

The dynamic relationship between predator and its prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. The traditional predator–prey models have been studied extensively (see, for example, [1–5] and references cited therein).

Recently, there is a growing explicit biological and physiological evidence [6–8] that in many situations, especially when predators have to search for food (and, therefore, have to share or compete for food), a more suitable general predator–prey theory should be based on the so-called ratio-dependent theory, which can be roughly stated as that the per capita predator growth rate should be a function of the ratio of prey to predator abundance, and so should be the so-called ratio-dependent functional response. This is strongly supported by numerous field and laboratory experiments and observations [9,10].

Based on the Michaelis–Menten or Holling type-II function, Arditi and Ginzburg [11] proposed a ratio-dependent function of the form

$$P\left(\frac{x}{y}\right) = \frac{c\left(\frac{x}{y}\right)}{m + \left(\frac{x}{y}\right)} = \frac{cx}{my + x}$$

and the following ratio-dependent predator–prey model:

$$\begin{cases} x' = x(a - bx) - \frac{cxy}{my+x}, \\ y' = y\left(-d + \frac{fx}{my+x}\right). \end{cases} \tag{1.1}$$

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Here $x(t)$ and $y(t)$ represent the densities of the prey and the predator at time t, respectively. a/b is the carrying capacity, $d > 0$ is the death rate of the predator, and a, c, m and f/c are positive constants that stand for the intrinsic growth rate of the prey, capturing rate, half saturation constant and conversion rate of the predator, respectively. Later on, Beretta and Kuang [12], Hsu et al. [6], Jost et al. [7], and Kuang and Beretta [8] investigated system (1.1).

The ratio-dependent predator–prey models with or without time delays have been studied by many researchers recently and very rich dynamics have been observed (see, for example, [13–24] and references cited therein).

In view of periodicity of the actual environment, Fan and Wang [24] established verifiable criteria for the global existence of positive periodic solutions of a more general delayed ratio-dependent predator–prey model with periodic coefficients of the form

$$\begin{cases} x'(t) = x(t)\left[a(t) - b(t)\int_{-\infty}^{t} k(t-s)x(s)\mathrm{d}s\right] - \frac{c(t)x(t)y(t)}{my(t)+x(t)}, \\ y'(t) = y(t)\left[-d(t) + \frac{f(t)x(t-\tau(t))}{my(t-\tau(t))+x(t-\tau(t))}\right]. \end{cases} \tag{1.2}$$

By substituting Holling type III functional response for Holling type II functional response in system (1.2), Wang and Li [23] studied a delayed ratio-dependent predator–prey model with Holling type III functional response of the form

$$\begin{cases} x'(t) = x(t)\left[a(t) - b(t)\int_{-\infty}^{t} k(t-s)x(s)\mathrm{d}s\right] - \frac{c(t)x^2(t)y(t)}{m^2y^2(t)+x^2(t)}, \\ y'(t) = y(t)\left[-d(t) + \frac{f(t)x^2(t-\tau)}{m^2y^2(t-\tau)+x^2(t-\tau)}\right]. \end{cases} \tag{1.3}$$

In 1991, Kuang [25] studied the local stability and oscillation of the following neutral delay Gause-type predator–prey system

$$\begin{cases} x'(t) = rx(t)\left[1 - \frac{x(t-\tau)+\rho x'(t-\tau)}{K}\right] - y(t)p(x(t)), \\ y'(t) = y(t)[-\alpha + \beta p(x(t-\sigma))]. \end{cases} \tag{1.4}$$

As pointed out by Freedman and Wu [26] and Kuang [27], it would be of interest to study the existence of periodic solutions for periodic systems with time delay. The periodic solutions play the same role played by the equilibria of autonomous systems. In addition, in view of the fact that many predator–prey systems display sustained fluctuations, it is thus desirable to construct predator–prey models capable of producing periodic solutions.

In this paper, motivated by the above work, we shall study the existence of positive periodic solutions for the following neutral delay ratio-dependent predator–prey model with Holling type III functional response

$$\begin{cases} x'(t) = x(t)[r(t) - a(t)x(t-\sigma_1) - \rho x'(t-\sigma_2)] - \frac{c(t)x^2(t)}{m^2y^2(t)+x^2(t)}y(t), \\ y'(t) = y(t)\left[-d(t) + \frac{h(t)x^2(t-\tau(t))}{m^2y^2(t-\tau(t))+x^2(t-\tau(t))}\right]. \end{cases} \tag{1.5}$$

For convenience, we will use the notations:

$$|f|_0 = \max_{t\in[0,\omega]}\{|f(t)|\}, \quad \bar{f} = \frac{1}{\omega}\int_0^{\omega} f(t)\mathrm{d}t, \quad \hat{f} = \frac{1}{\omega}\int_0^{\omega} |f(t)|\mathrm{d}t,$$

where $f(t)$ is a continuous ω–periodic function. In this paper, we always make the following assumptions for system (1.5).

(H_1) $\rho > 0,\ m > 0,\ \sigma_1,\ \sigma_2$ are four constants. $\tau(t),\ r(t),\ d(t),\ a(t),\ c(t),\ h(t)$ are continuous ω-periodic functions. In addition, $\bar{r} > 0,\ \bar{d} > 0,\ a(t) > 0,\ c(t) > 0,\ h(t) > 0$ for any $t \in [0,\omega]$;
(H_2) $\rho e^B < 1$, where $B = \ln A + \rho A + (\hat{r} + \bar{r})\omega$ and $A = \max_{t\in[0,\omega]}\left\{\frac{2\bar{r}}{a(t)}\right\}$.
(H_3) $\bar{c} < 2m\bar{r}$.
(H_4) $\bar{d} < \bar{h}$.

Our aim in this paper is, by using the coincidence degree theory developed by Gaines and Mawhin [28], to derive a set of easily verifiable sufficient conditions for the existence of positive periodic solutions of system (1.5).

2. Existence of positive periodic solution

In this section, we shall study the existence of at least one positive periodic solution of system (1.5). The method to be used in this paper involves the applications of the continuation theorem of coincidence degree. For the readers' convenience, we introduce a few concepts and results about the coincidence degree as follows.

Let X, Z be real Banach spaces, $L : \mathrm{Dom}L \subset X \to Z$ be a linear mapping, and $N : X \to Z$ be a continuous mapping.

The mapping L is said to be a Fredholm mapping of index zero, if $\dim\mathrm{Ker}L = \mathrm{codim}\,\mathrm{Im}L < +\infty$ and $\mathrm{Im}L$ is closed in Z.

If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$, such that $\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Ker}Q = \mathrm{Im}L = \mathrm{Im}(I - Q)$. It follows that the restriction L_P of L to $\mathrm{Dom}L \cap \mathrm{Ker}P : (I - P)X \to \mathrm{Im}L$ is invertible. Denote the inverse of L_P by K_P .