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Numerical solution of the MHD Reynolds equation for squeeze film lubrication between two parallel surfaces

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ABSTRACT

Characteristic features of squeeze film lubrication between two rectangular plates, of which, the upper plate has a roughness structure, in the presence of a uniform transverse magnetic field are examined. The fluid in the film region is represented by a viscous, incompressible and electrically conducting couple-stress fluid. The thickness of the fluid film region is h and that of the roughness is h_s. The pressure distribution in the film region is governed by the modified Reynolds equation, which also incorporates the roughness structure and couple stress fluid. This Reynolds equation is solved using a novel multigrid method for all involved physical parameters. It is observed that the pressure distribution, load carrying capacity and squeeze film-time increase for smaller values of couple-stress parameter and for increasing roughness parameters and Hartmann number compared to the classical case.

1. Introduction

The hydrodynamic lubrication theory for rough surfaces has been studied with considerable interest in recent years, because, all bearing surfaces are rough to some extent. All bearing surfaces develop roughness after having some run-in and wear. In some cases, contamination of lubricant is also one of the reasons to generate surface roughness through chemical degradation. Several approaches have been proposed in the literature to study the effect of surface roughness on bearing surfaces. Burton [1] modeled roughness by a Fourier series type approximation. Since, the surface roughness distribution is random in nature, a stochastic approach has to be adopted. Thus, Christensen [2] developed a stochastic theory for the study of rough surfaces in hydrodynamic lubrication. Since then many researchers have adopted and used extensively this approach to study roughness effect on bearing surfaces. For example, Prakash and Tiwari [3] used this theory to study the effect of surface roughness on the porous bearings. Gururajana and Prakash [4] have successfully used this theory to study the influence of roughness on narrow journal bearing in which bearing has a porous material and showed that roughness increases the pressure distribution in the fluid film region. Chiang et al. [5] have analysed the lubrication performance of rough finite journal bearings. Bujurke and Kudenatti [6] have numerically solved the modified Reynolds equation to explore the effects of surface roughness on articular cartilages in synovial joints by modeling them as the load sustaining bearings and showed that the pressure rise and load carrying capacity are more compared to the classical case.

The magnetohydrodynamic (MHD) flow of a fluid in a squeeze film lubrication is of interest, because, it prevents the unexpected variation of lubricant viscosity with temperature under sever operating conditions and also investigations into

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the effects of magnetic field in lubrication have been encouraging because of their importance in engineering applications, with obvious relevance to technology based world. The MHD lubrication in an externally pressurized thrust bearing has been investigated both theoretically and experimentally by Maki et al. [7]. Copious papers on MHD lubrication are available in the literature which include- MHD slider bearings ([8], [9]), MHD Journal bearings ([10], [11]), MHD squeeze film bearings ([12]). Hamza [13] has showed the effects of MHD on a fluid film squeezed between two rotating surfaces. Bujurke and Kudenatti [14] have theoretically explored the effect of roughness on the electrically conducting fluid in the rectangular plates, in which upper plate has a roughness structure. They modified the classical Reynolds equation to include the effects of roughness and magnetic field and solved it using a multigrid method. They showed that the effect of roughness and Hartmann number is to increase the pressure distribution and hence the load carrying capacity for increasing roughness and magnetic parameters. Hsu et al. [15] have studied MHD effects in circular disks with inertial effects and showed that the bearing characteristics are more pronounced for applied magnetic field.

Most of the theoretical investigations mentioned above, have been carried out by assuming the lubricant between two surfaces is Newtonian. Results of Newtonian fluid give a satisfactory understanding but this theory fails to give the effect of the non-Newtonian fluid. However, with development of modern industries, the importance of non-Newtonian fluid as a lubricant when squeeze film takes place, is important for the most of engineering applications. So, the effect of non-Newtonian fluid must be taken into account in the study of bearings. Polymer-thickened oils, greases, synovial fluid etc. are simple examples of non-Newtonian fluid. Stokes [16] introduced a microcontinuum theory for couple-stress fluids which is the simplest generalization of classical theory, and also accounts for polar effects such as presence of anti-symmetrical stresses, couple stresses and body couples etc. Naduvinamani et al. [17,18] have extensively studied the effect of couple-stress fluid on lubrication characteristics of various bearings such as journal and squeeze film bearings and showed that pressure distribution and load carrying capacity are more pronounced for couple-stress fluid compared to the Newtonian fluid. Lin and Hung [19] have investigated the combined effects of non-Newtonian couple-stress fluid and rotational inertia on the squeeze film behavior between rotating circular discs and showed that couple-stress fluid increases the load capacity and squeezing time of the bearing.

Thus, keeping the above discussion in mind, we investigate the effects of non-Newtonian couple-stress fluid and roughness on hydrodynamic squeeze film mechanisms in the presence of transverse magnetic field which have useful applications in understanding bearing characteristics.

Rest of the paper has been organized as follows. The required basic equations supplemented with appropriate boundary conditions are given and subsequently modified MHD Reynolds equation is derived in Section 2. To analyze the effect of roughness and couple-stress fluid, the Reynolds equation for longitudinal roughness is also derived in this section. In Section 3, the Reynolds equation is discretized with a finite difference method, and solved using multigrid method for fluid film pressure, load carrying capacity and squeeze film time. Predictions on bearing characteristics are given for varying couple-stress, roughness parameters, Hartmann number and aspect ratio in Section 4. Final section summarises the important findings and their usefulness in designing bearings.

2. Formulation of the problem

The model consists of flow of viscous isothermal and incompressible electrically conducting couple-stress fluid between two rectangular plates in which the upper plate has a roughness structure. The physical configuration of the problem is shown in Fig. 1. The upper rough plate approaches the lower smooth plate with a constant velocity $\frac{dH}{dt}$. A uniform transverse magnetic field M_0 is applied in the z-direction. The upper and lower plates are separated by thickness H, then, the total film thickness is made up of two parts as

$$H = h(t) + h_s(x, y, \xi), \tag{1}$$

where $h(t)$ is the height of the nominal smooth part of the film region, and h_s is part due to the surface asperities measured from the nominal level, which is a randomly varying quantity of zero mean, and ξ is the index parameter determining a definite roughness structure. In addition to the usual assumptions of lubrication theory, we assume fluid inertia is negligible, and except the Lorentz force, the body forces are also neglected. Under these assumptions, the governing equations in Cartesian co-ordinate system are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{2a}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} - \sigma M_0^2 u, \tag{2b}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} - \eta \frac{\partial^4 v}{\partial z^4} - \sigma M_0^2 v, \tag{2c}$$

$$\frac{\partial p}{\partial z} = 0, \tag{2d}$$