Applied Mathematics and Computation 218 (2012) 9997–10006

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Asymptotic and exponential stability of uncertain system with interval delay ☆

Liangdong Guo [a,*], Hong Gu [b], Jun Xing [a], Xiqin He [a]

[a] School of Science, Liaoning University of Science and Technology, Anshan 114051, PR China
[b] Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian 116024, PR China

ARTICLE INFO

Keywords:
Time-delay system
Exponential stability
Linear matrix inequality
Integral equality
Delay-range-dependence

ABSTRACT

This paper considers the problem of asymptotic and exponential stability analysis of the continuous system with interval time-varying delay and uncertainties. By constructing a new Lyapunov–Krasovskii functional, several delay-range-dependent conditions are derived in terms of the linear matrix inequalities (LMIs). Two novel integral equalities are employed to overcome the disadvantages of the methods in existing references. The merits of the proposed criteria lie in the exponential stability being a free value and the less conservativeness. Finally, some numerical examples are included to illustrate the effectiveness and the improvement of the proposed method.

1. Introduction

Time-delays are frequently encountered in many fields of science and engineering, including communication network, manufacturing systems, biology, economy and other areas. In practice, the systems almost present some uncertainties because it is very difficult to obtain an exact mathematical model due to environmental noise, uncertain or slowly varying parameters, etc. The existence of the time-delays and uncertainties are frequently encountered in various practical systems and very often are the causes for instability and poor performance of control systems existence is frequently a source of oscillation and instability [1–3]. Therefore, robust stability analysis and synthesis of continuous systems with delayed state are problems of recurring interest [4–12]. Nowadays, stability analysis is mainly concerned with two categories of stability conditions: delay-independent [4] and delay-dependent [5–12]. Delay-dependent stability criteria are less conservative than delay-independent ones, particularly when the delay is small because the former does not include any information on the size of delay while the latter employs such information. For delay-dependent type, much attention has been paid to reduce the conservatism of stability conditions, for example, generalized discretized Lyapunov–Krasovskii functional approach [5], free weighting matrix techniques [6], model transformation techniques [8] and argument Lyapunov–Krasovskii functionals method [10]. Some inequalities [4,13–18] were well used to deal with the bounding for the cross term. Generally speaking, the so-far discussed delay of the continuous systems can be classified into two types; that is, slow time-varying delay and fast time-varying delay. In general, the results for the slow time-varying delay case are often less conservative than those for the fast time-varying delay case due to the former taking advantage of the additional information of the delay. However, the delay-differential condition can yield conservatism.

Recently, there has been rapidly growing interest in the problem of analysis and synthesis for of delayed systems with interval time varying delays (i.e., the lower bound of the delay is 0) for the systems have strong background in engineering

☆ Supported by the National Natural Science Foundation of China under Grant 61174215.
* Corresponding author.
E-mail addresses: liangdongguo@foxmail.com (L. Guo), guhong@dlut.edu.cn (H. Gu).

0096-3003/$ - see front matter
http://dx.doi.org/10.1016/j.amc.2012.03.074

applications, in areas such as network congestion control and networked control systems [19–24]. For example, He et. al. [20] proposed a new stability criterion for uncertain systems with interval time-varying delay by introducing augmented Lyapunov functional and considering the additional useful term which are ignored in previous methods; in [21], the authors have studied robust stability of the system with interval time-varying delay and norm bounded parameter uncertainty; to avoid the "over bounding" for a cross term, the improved robust stability criteria of the system were proposed based on a novel augmented Lyapunov functional in [22]. Shao [24] provided a new delay-dependent stability criterion for systems with a delay varying in an interval by utilizing the convex combination method. In fact, the similar method was also employed in [25,26]. However, there is still room for further improvements of the stability criteria. For instance, when using inequalities in [4,13–18] to deal with the bounding for the terms $\int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s)Z\dot{x}(s)ds$ and $\int_{t-\tau(t)}^{t-\tau_m} \dot{x}^T(s)Z\dot{x}(s)ds$, both $\tau_M - \tau(t)$ and $\tau(t) - \tau_m$ were enlarged as $\tau_M - \tau_m$, that is, $\tau(t)$ was seen as $(\tau_M + \tau_m)/2$, which leads to conservativeness. In fact, the similar conservative estimation exists in [17,20,24–26].

It is worth noting that the above literature provided the conditions for asymptotic stability. In fact, it is interesting and important to find estimates of the exponential bounds for solutions of uncertain time-delay systems because it is also important to get the convergence rates of prescribed time-delay systems [17,27–32]. In [28,30], some exponential conditions were propose by using Lyapunov–Krasovskii functional method and LMI technique. In Kwon and Park [17], a delay-dependent exponential stability criterion for uncertain dynamic systems with time-varying delays based on the Lyapunov function method and an integral inequality, which proved to be less conservative than the one in [28,30] in finding the maximum delay bounds. In [32] the robust stability conditions are derived in terms of LMIs based on an improved Lyapunov–Krasovskii functional combined with Leibniz–Newton formula, which allows to compute simultaneously the two bounds that characterize the exponential stability rate of the solution. However, the exponential stability for uncertain dynamic systems with interval time-varying delays, which is more general case than time-varying one, is not considered in these works.

Inspired by the above discuss, we will consider the robust exponential stability of uncertain continuous systems with interval time-varying delay in this work. The time-varying delay is assumed to belong to an interval and a fast time-varying function. The uncertainties are considered to be norm-bounded. To overcome the aforementioned disadvantages of the methods, we firstly propose two new bounding integral equalities. Then, based on a modified Lyapunov–Krasovskii functional and the integral equalities, several delay-range-dependent exponential stability criteria are derived in term of the linear matrix inequalities. As a by-product of the exponential criterion, some robust asymptotic stability conditions are obtained. Compared with the conditions obtained in the related existing literature, the merit of the proposed one lies in the less conservativeness. Finally, some numerical examples are illustrated to show the improvement of this paper.

Notation: the superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of all real matrices, the notation $P > 0(\geqslant 0)$ for means that is symmetric and positive (semi-positive) definite. I denotes the identity matrix of appropriate dimensions. $\lambda_{min}(A)$ and $\lambda_{max}(A)$ denote the minimum and maximum eigenvalue of the real symmetric matrix A, respectively. The shorthand diag$\{M_1, M_2, \ldots, M_n\}$ denotes a block-diagonal matrix with diagonal blacks being the matrices $M_1, M_2, \ldots, M_n$. In addition, in symmetric block matrices or long matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Preliminaries

Consider the uncertain continuous-time systems as follows

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - \tau(t)), \\ x(t_0 + \theta) = \phi(t), \quad t \in [-\tau_M, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $\phi(t)$ is the initial condition of the system; $A, B, \in \mathbb{R}^{n\times n}$ are known constant real matrices; $\tau(t)$ is the time varying delay and assumed to satisfy

$$0 < \tau_m \leqslant \tau(t) < \tau_M. \tag{2}$$

The uncertainties $\Delta A(t), \Delta B(t)$ are time-varying matrices with appropriate dimensions, which are defined as follows:

$$[\Delta A(t), \quad \Delta B(t)] = D\Sigma(t)[F_1, F_2], \tag{3}$$

where D, F_1, F_2 are known constant matrices; $\Sigma(t) \in \mathbb{R}^{n\times n}$ is unknown real time-varying matrices with Lebesgue measurable elements bounded by

$$\Sigma^T(t)\Sigma(t) \leqslant I. \tag{4}$$

Therefore, system (1) with uncertainties satisfying (3) and (4) can be rewritten in the following forms:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t - \tau(t)) + Dp(t), \\ p(t) = \Sigma(t)q(t), \\ q(t) = F_1x(t) + F_2x(t - \tau(t)), \\ x(t_0 + \theta) = \phi(t), \quad t \in [-\tau_M, 0]. \end{cases} \tag{5}$$