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An EOQ model for salesmen's initiatives, stock and price sensitive demand of similar products – A dynamical system

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ABSTRACT

The paper deals with an inventory model to determine the retailer's optimal order quantity for similar products. It is assumed that the amount of display space is limited and the demand of the products depends on the display stock level where more stock of one product makes a negative impression of the another product. Besides it, the demand rate is also dependent on selling price and salesmen's initiatives. Also, the replenishment rate depends on the level of stock of the items. The objective of the model is to maximize the profit function, including the effect of inflation and time value of money by Pontryagin's Maximal Principles. The stability analysis of the concerned dynamical system has been done analytically.

1. Introduction

Most of the management scientist and industrial engineers have recognized the influence of displayed stock level on customers. According to Levin et al. [1], "large piles of consumer goods displayed in a supermarket will lead the customer to buy more". Silver and Peterson [2] also noted that sales at the retail level tend to be proportional to the amount of inventory displayed. Baker and Urban [3] considered a power-form inventory-level-dependent demand rate that would decline along with the stock level throughout the entire cycle. Ghosh and Chaudhuri [4] developed an order level inventory model for a deteriorating item with two levels of storage stock-dependent demand pattern. Zhou et al. [5] considered a decentralized two-echelon supply chain where the demand of the product is dependent on the inventory-level on display. Soni and Shah [6] formulated an optimal ordering policies for retailer in trade credit environment where demand was partially constant and partially dependent on the stock. Goyal and Chang [7] developed an ordering-transfer inventory model when the storage capacity is limited and the demand rate depends on display stock level. They obtained the retailer's optimal ordering quantity and the number of transfer per order from the warehouse to the display area for maximizing the average profit per unit yielded by the retailer. Hsieh and Dye [8] provided some useful properties for finding the optimal replenishment schedule with stock-dependent demand under exponential partial backlogging. Min et al. [9] investigated an EOQ model for perishable items under permissible delay in payments where demand of the products varies linearly with the level of stock. Sajadieh et al. [10] developed an integrated vendor–buyer model for a two-stage supply chain, assuming demand of the products as a positive power function of displayed inventory. The research articles related to the stock-dependent demand pattern are Urban [11], Pal et al. [12], Goh [13], Padmanabhan and Vrat [14], Sarker et al. [15], Datte and Paul [16], Balkhi and Benkherouf [17], Chang [18], Hou and Lin [19], Min and Zhou [20], among others.

In recent years, many businesses have been involved in various forms of promotional effort to boost market demand. Goyal and Gunasekaran [21] presented an integrated production–inventory–marketing model for determining the EPQ

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(economic production quantity) and EOQ (economic order quantity) for raw materials in a multi-stage production system. They considered the effect of different marketing policies such as the price per unit product and the advertisement frequency on the demand of a perishable item. Many studies have focused on the effect of promotions on sales using store or market level data (Dube [22]; Naik et al. [23]; Divakar et al. [24]). Sun [25] made relation of the customers behavior with different types of promotions and identified that promotions had a strong impact on stronger brands. Based on the work of Divakar et al. [24], Ramanathan and Muyldermans [26] applied structural equation modeling to investigate the impact of promotions and other factors on the sales of soft drinks. Sana and Chaudhuri [27] developed an inventory model for stock with advertising sensitive demand. Sana [28] investigated an interesting multi-item EOQ model for deteriorated and ameliorating items when the time varying demand is influenced by enterprizes' initiatives like advertising media and salesmen' effort. In a competitive market, the quality of a product and fair price in any business organization play an important role in marketing management. The pricing of sales commodities is a focal point in any business organization in a given economy. Sana [29] formulated an EOQ model over an infinite time horizon for deteriorating items while the demand is price-sensitive, considering partial backordering and time dependent deterioration rate. Recently, Khanra et al. [30] investigated an EOQ model for a deteriorating item with time dependent quadratic demand under permissible delay in payment.

The monetary situation in each countries has changed to such an extent due to large scale inflation, and consequent sharp declines in the purchasing power of money. So, it has not been possible to ignore the effects of inflation and time value of money. Buzacott [31] was the first in this direction who derived expressions for the optimal order quantity, considering inflation and time value of money. In this direction, the works of Biermann and Thomas [32], Datta and Pal [33], Bose et al. [34] are worth mentioning, among others.

In this paper, a dynamical system of differential equation of two types of similar products has been considered when the demand rate of each product depends on the salesmen' initiatives and level of stock of the products and its selling prices. The replenishment rates of the products are also dependent on the on-hand inventory. The huge stock of one product decreases the demand of the another product. The stability analysis of the system has been done. Finally, a profit function considering inventory cost, purchasing price, cost of effort and sales prices has been optimized by Pontryagin's Maximal Principles.

2. Notation

The following notation are considered to develop the model:
Notation:

$X(t)$ – on hand stock of item 1 at time 't'.
$Y(t)$ – on hand stock of item 2 at time 't'.
D_x – demand rate of item 1.
D_y – demand rate of item 2.
R_x – replenishment rate of item 1.
R_y – replenishment rate of item 2.
L_x – maximum storage capacity of item 1.
L_y – maximum storage capacity of item 2.
L – maximum storage capacity ($L = L_x + L_y$) of item 1 and item 2.
$E(t)$ – joint effort function of the advertising and salesmen' initiatives at time 't'.
p_x – purchasing cost per unit item 1.
p_y – purchasing cost per unit item 2.
h_x – inventory holding cost per unit item 1 per unit time.
h_y – inventory holding cost per unit item 2 per unit time.
γ – cost per unit effort.
S_x – selling price per unit item 1.
S_y – selling price per unit item 2.
S_x^{max} – upper bound of selling price per unit item 1 at which demand is zero.
S_y^{max} – upper bound of selling price per unit item 2 at which demand is zero.
$\delta = (r - i)$ – r and i are rate of interest and inflation per unit currency.

3. Formulation of the model

The author considers a stock-dependent inventory model of two similar products such that the demand of the products depends on the on-hand inventory ($X(t), Y(t)$), selling price ($S_x(t), S_y(t)$) and effort ($E(t)$) by advertising or salesmen' initiatives. As the demand of the products are dependent on the level of stock, the replenishment rate varies with the stock-level of the products. Here, the demand rates of item 1 and item 2 are as follows:

$$D_x = \frac{C_x E X(1 - Y/L)}{l_1 E + l_2 X}, \tag{1}$$