Applied Mathematics and Computation 218 (2011) 3399–3413

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Comparison results of the preconditioned AOR methods for L-matrices ☆

Jae Heon Yun

Department of Mathematics, College of Natural Sciences, Chungbuk National University, Cheongju, Chungbuk 361-763, Republic of Korea

ARTICLE INFO

Keywords:
Preconditioned AOR (PAOR) method
L-matrix
Preconditioner
Krylov subspace method

ABSTRACT

In this paper, we first provide comparison results of several types of the preconditioned AOR (PAOR) methods for solving a linear system whose coefficient matrix is an L-matrix satisfying some weaker conditions than those used in the recent literature. Next, we propose an application of PAOR method to a preconditioner of Krylov subspace method. Lastly, numerical results are provided to show that Krylov subspace method with the PAOR preconditioner performs quite well as compared with the ILU (0) preconditioner.

© 2011 Elsevier Inc. All rights reserved.

1. Introduction

In this paper, we consider the following linear system

$$Ax = b, \quad x, b \in \mathbb{R}^n, \tag{1}$$

where $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ is a nonsingular matrix. The basic iterative method for solving the linear system (1) can be expressed as

$$x_{k+1} = M^{-1}Nx_k + M^{-1}b, \quad k = 0, 1, \ldots, \tag{2}$$

where x_0 is an initial vector and $A = M - N$ is a splitting of A. $T = M^{-1}N$ is called the iteration matrix of the basic iterative method (2).

Throughout the paper, we assume that $A = I - L - U$, where I is the identity matrix, and L and U are strictly lower triangular and strictly upper triangular matrices, respectively. Then the iteration matrix of the AOR iterative method [5] for solving the linear system (1) is

$$T_{r,\omega} = (I - rL)^{-1}((1-\omega)I + (\omega - r)L + \omega U), \tag{3}$$

where ω and r are real parameters with $\omega \neq 0$.

In order to accelerate the convergence of iterative method for solving the linear system (1), the original linear system (1) is transformed into the following preconditioned linear system

$$PAx = Pb, \tag{4}$$

where P, called a *preconditioner*, is a nonsingular matrix. Wang and Song [14] presented a general form of the preconditioners P for nonsingular M-matrices. In this paper, we consider the following four types of preconditioners $P = P_k$ ($1 \leqslant k \leqslant 4$): the preconditioner P_1 is of the form $P_1 = I + S_1$, where

☆ This work was supported by the Korea Research Foundation (KRF) Grant funded by the Korea Government (MEST) (Nos. 2009-0072541 and 2010-0016538).

E-mail address: gmjae@chungbuk.ac.kr

0096-3003/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2011.08.085

$$S_1 = \begin{pmatrix} 0 & -\alpha_1 a_{12} & 0 & \cdots & 0 \\ 0 & 0 & -\alpha_2 a_{23} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\alpha_{n-1} a_{n-1,n} \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}.$$

The preconditioner P_2 is of the form $P_2 = I + S_2$, where

$$S_2 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\alpha_n a_{n1} & 0 & \cdots & 0 \end{pmatrix}.$$

The preconditioner P_3 is of the form $P_3 = I + S_3$, where

$$S_3 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ -\alpha_2 a_{21} & 0 & \cdots & 0 \\ -\alpha_3 a_{31} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -\alpha_n a_{n1} & 0 & \cdots & 0 \end{pmatrix}.$$

The preconditioner P_4 is of the form $P_4 = I + S_4$, where

$$S_4 = \begin{pmatrix} 0 & 0 & \cdots & -\alpha_1 a_{1n} \\ 0 & 0 & \cdots & -\alpha_2 a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -\alpha_{n-1} a_{n-1,n} \\ 0 & 0 & \cdots & 0 \end{pmatrix}.$$

The preconditioner P_1 was first introduced by Gunawardena et al. [4] when $\alpha_i = 1$ $(1 \leqslant i \leqslant n-1)$, and it has been studied by Kohno et al. [7] and Wu et al. [15] for $0 \leqslant \alpha_i \leqslant 1$ $(1 \leqslant i \leqslant n-1)$. The preconditioner P_2 was first introduced by Evans et al. [3] when $\alpha_n = 1$, and it has been studied by Yun [17] and Li et al. [8] when $0 < \alpha_n \leqslant 1$. The preconditioner P_3 was first introduced by Milaszewicz [10] when $\alpha_i = 1$ $(2 \leqslant i \leqslant n)$, and it has been studied by Yun [16,19] for $\alpha_i = 1$ $(2 \leqslant i \leqslant n)$ and Huang et al. [6] for $0 \leqslant \alpha_i \leqslant 1$ $(2 \leqslant i \leqslant n)$. The preconditioner P_4 was recently introduced by Dehghan and Hajarian [2].

Let $A_1 = P_1 A$ and $S_1 L = E_1 + F_1$, where E_1 is a diagonal matrix and F_1 is a strictly lower triangular matrix. Then one obtains

$$A_1 = (I + S_1)(I - L - U) = I - L - U + S_1 - S_1 L - S_1 U = D_1 - L_1 - U_1, \tag{5}$$

where $D_1 = I - E_1$, $L_1 = L + F_1$, and $U_1 = U - S_1 + S_1 U$.

Let $A_2 = P_2 A$ and $S_2 U = E_2 + F_2$, where E_2 is a diagonal matrix and F_2 is a strictly lower triangular matrix. Using $S_2 L = 0$, one obtains

$$A_2 = (I + S_2)(I - L - U) = I - L - U + S_2 - S_2 U = D_2 - L_2 - U_2, \tag{6}$$

where $D_2 = I - E_2$, $L_2 = L - S_2 + F_2$, and $U_2 = U$.

Let $A_3 = P_3 A$ and $S_3 U = E_3 + F_3 + G_3$, where E_3 is a diagonal matrix, F_3 is a strictly lower triangular matrix and G_3 is a strictly upper triangular matrix. Using $S_3 L = 0$, one obtains

$$A_3 = (I + S_3)(I - L - U) = I - L - U + S_3 - S_3 U = D_3 - L_3 - U_3, \tag{7}$$

where $D_3 = I - E_3$, $L_3 = L - S_3 + F_3$, and $U_3 = U + G_3$.

Let $A_4 = P_4 A$ and $S_4 L = E_4 + F_4 + G_4$, where E_4 is a diagonal matrix, F_4 is a strictly lower triangular matrix and G_4 is a strictly upper triangular matrix. Using $S_4 U = 0$, one obtains

$$A_4 = (I + S_4)(I - L - U) = I - L - U + S_4 - S_4 L = D_4 - L_4 - U_4, \tag{8}$$

where $D_4 = I - E_4$, $L_4 = L + F_4$, and $U_4 = U - S_4 + G_4$.

If we apply the AOR iterative method to the preconditioned linear systems (4), then we get the *preconditioned AOR iterative method* whose iteration matrix is

$$T_{k,r,\omega} = (D_k - rL_k)^{-1}((1-\omega)D_k + (\omega - r)L_k + \omega U_k) \quad \text{if } P = P_k (1 \leqslant k \leqslant 4). \tag{9}$$

If $\omega = r$, then the AOR method and the preconditioned AOR method reduce to the SOR method and the preconditioned SOR method, respectively.

Download English Version:

https://daneshyari.com/en/article/4630345

Download Persian Version:

https://daneshyari.com/article/4630345

Daneshyari.com