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Permanence of a discrete nonlinear N-species cooperation system with time delays and feedback controls

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ABSTRACT

A discrete nonlinear N-species cooperation system with time delays and feedback controls is considered in this paper. Sufficient conditions which ensure the permanence of the system are obtained. It is shown that these conditions are weaker than those of Chen [F.D. Chen, Permanence of a discrete N-species cooperation system with time delays and feedback controls, Appl. Math. Comput. 186(2007) 23–29], that is, our investigation shows that the additional condition in Chen's paper is not necessary.

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1. Introduction

The aim of this paper is to investigate the permanence of the following non-autonomous discrete nonlinear N-species cooperation system with time delays and feedback controls

$$x_i(n+1) = x_i(n)\exp\left\{r_i(n)\left[1 - \frac{x_i(n-\tau_{ii})}{a_i(n)+\sum_{j=1,j\neq i}^{n} b_{ij}(n) f_j(x_j(n-\tau_{ij}))} - c_i(n)x_i(n-\tau_{ii})\right] - d_i(n)u_i(n) - e_i(n)u_i(n-\eta_i)\right\},$$

$$\Delta u_i(n) = -\alpha_i(n)u_i(n) + \beta_i(n)x_i(n) + \gamma_i(n)x_i(n-\sigma_i), \tag{1.1}$$

where $x_i(n)$ $(i = 1,2,\ldots,n)$ is the density of cooperation species i at kth generation, $u_i(n)$ is the control variable, Δ is the first-order forward difference operator $\Delta u_i(n) = u_i(n+1) - u_i(n)$. For more background and biological adjustments of system (1.1), one could refer to [1–14] and the references cited therein.

Throughout this paper, we assume that

(H) $\alpha_i(n): Z \to (0,1)$; bounded sequences $r_i(n)$, $a_i(n)$, $b_i(n)$, $c_i(n)$, $d_i(n)$, $e_i(n)$, $\beta_i(n)$ and $\gamma_i(n): Z \to R^+$; $f_j: R^+ \to R^+$; $f_j'(x_j) \geqslant 0$; τ_{ij}, τ_{ii} and σ_i are positive integer; Z, R^+ denote the sets of all integers and all positive real numbers, respectively; $\tau = \max\{\max_{1\leqslant i,j\leqslant n}\tau_{ij}, \max_{1\leqslant i\leqslant n}\sigma_i\} > 0$.

It is not difficult to prove that $x_i(n) > 0$, $u_i(n) > 0$ for all $i = 1,\ldots,n$.

If $f_j(x_j(n-\tau_{ij})) = x_j(n-\tau_{ij})$, system (1.1) reduces to the following system

$$x_i(n+1) = x_i(n)\exp\left\{r_i(n)\left[1 - \frac{x_i(n-\tau_{ii})}{a_i(n)+\sum_{j=1,j\neq i}^{n} b_{ij}(n) x_j(n-\tau_{ij})} - c_i(n)x_i(n-\tau_{ii})\right] - d_i(n)u_i(n) - e_i(n)u_i(n-\eta_i)\right\},$$

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$$\Delta u_i(n) = -\alpha_i(n)u_i(k) + \beta_i(n)x_i(n) + \gamma_i(n)x_i(n-\sigma_i), \tag{1.2}$$

Throughout this paper, for any bounded sequence $\{a(n)\}$,

$$a^u = \sup_{n\in N} a(n), \quad a^l = \inf_{n\in N} a(n).$$

We say that system (1.1) is permanent if there are positive constants M and m such that for each positive solution $(x_1(n),\ldots,x_n(n),u_1(n),\ldots,u_n(n))$ of system (1.1) satisfies

$$m \leqslant \lim_{n\to+\infty} \inf x_i(n) \leqslant \lim_{n\to+\infty} \sup x_i(n) \leqslant M,$$
$$m \leqslant \lim_{n\to+\infty} \inf u_i(n) \leqslant \lim_{n\to+\infty} \sup u_i(n) \leqslant M.$$

for all $i = 1,2,\ldots,n$.

Chen [2] considered the above system (1.2). Assume that (H) and

$$r_i > (d_i^u + e_i^u)M_{i2}, \quad i = 1,2,\ldots,n, \tag{1.3}$$

hold, then the system (1.2) is permanent, where

$$M_{i2} = \frac{(\beta_i^u + \gamma_i^u)\exp\{r_i^u(\tau_{ii}+1)-1\}}{\alpha_i^l c_i^l r_i^u}, \quad i = 1,2,\ldots,n.$$

The aim of this paper is, by further developing the analysis technique of Chen [13], to obtain sufficient conditions which ensure the permanence of the system (1.1). On the other hand, We should point out that conditions (H) are sufficient for the permanence of system (1.1) and (1.2). We found that if we use the method of Chen [13], then the additional condition, to some extent, is necessary. But for the system itself, this condition may not be necessary. Motivated by the above problem, we discuss the permanence of system (1.2) again, our investigation shows that condition (1.3) is also not necessary.

2. Permanence

In this section, we establish a permanence results for system (1.1) and (1.2). In order to prove our main result, firstly we give some lemmas which will be useful for the following discussion.

Lemma 2.1 (*see* [5]). *Assume that $A > 0$ and $y(0) > 0$, and further suppose that*

(1)

$$y(n+1) \leqslant Ay(n) + B(n), \quad n = 1,2,\ldots \tag{2.1}$$

Then for any integer $k \leqslant n$,

$$y(n) \leqslant A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1). \tag{2.2}$$

Especially, if $A < 1$ and B is bounded above with respect to M, then

$$\lim_{n\to+\infty} \sup y(n) \leqslant \frac{M}{1-A}. \tag{2.3}$$

(2)

$$y(n+1) \geqslant Ay(n) + B(n), \quad n = 1,2,\ldots \tag{2.4}$$

Then for any integer $k \leqslant n$,

$$y(n) \geqslant A^k y(n-k) + \sum_{i=0}^{k-1} A^i B(n-i-1). \tag{2.5}$$

Especially, if $A < 1$ and B is bounded below with respect to m, then

$$\lim_{n\to+\infty} \inf y(n) \geqslant \frac{m}{1-A}. \tag{2.6}$$

Lemma 2.2 (*see* [15]). *Let $n \in N_{n_0}^+ = \{n_0, n_0+1,\ldots,n_0+l,\ldots\}$, $r \geqslant 0$. For any fixed n, $g(n,r)$ is a non-decreasing function with respect to r, and for $n \geqslant n_0$, following inequalities hold: $y(n+1) \leqslant g(n,y(n)), u(n+1) \geqslant g(n,u(n))$. If $y(n_0) \leqslant u(n_0)$, then $y(n) \leqslant u(n)$ for all $n \geqslant n_0$.*

Now let us consider the following single species discrete model:

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