Applied Mathematics and Computation 218 (2011) 1467–1477

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

An analytical method for magnetothermoelastic analysis of functionally graded hollow cylinders

Hong-Liang Dai [a,b,*], Yan-Ni Rao [b], Hao-Jie Jiang [b]

[a] State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha 410082, China
[b] Department of Engineering Mechanics, College of Mechanical and Vehicle Engineering, Hunan University, Changsha 410082, China

ARTICLE INFO

Keywords:
Functionally graded material (FGM)
Magnetothermoelastic
Hollow cylinder
Perturbation of magnetic field vector

ABSTRACT

This paper considers magnetothermoelastic behavior of a functionally graded material (FGM) hollow cylinder, placed in a uniform magnetic field, subjected to thermal and mechanical loads. Exact solutions for stresses and perturbations of the magnetic field vector in FGM hollow cylinders is determined by using the infinitesimal theory of magnetothermoelasticity. Numerical results indicate that the inhomogeneous constants presented in the present study are useful parameters from a design point of view in that it can be tailored for specific applications to control the stress and perturbation of magnetic field vector distributions. This research is helpful for the optimum design annular cylindrical FGM sensors/actuators.

1. Introduction

FGMs have been developed for structural components or mechanical elements such as those used in nuclear, aircraft and space engineering. To aid in the design of FGMs, it will be useful to have an understanding of the manner in which the inhomogeneous constants affect the induced magnetothermoelastic stresses and perturbation of magnetic field vector. To our knowledge, investigations on the exact solutions for the FGM hollow cylinder, placed in a uniform magnetic field, subjected to thermal and mechanical loads have not been found in literatures.

To date there have only been a few studies on mechanical behaviors of the FGM hollow cylinder. Zimmerman and Lutz [1] investigated the thermal stresses and effective thermal expansion in a uniformly heated FGM cylinder. Using the infinitesimal theory of elasticity, Naki and Murat [2] obtained a closed-form solution for stresses and displacements in functionally graded cylindrical and spherical vessels subjected to internal pressure. Wu et al. [3] presented an analytical study for piezothermoelastic behavior of a functionally graded piezoelectric cylindrical shell subjected to axisymmetric thermal of mechanical loading. Using the state space formulations, Chen et al. [4] investigated the free vibration of an arbitrarily thick orthotropic piezoelectric hollow cylinder with a functionally graded property along the thickness direction and filled with a non-viscous compressible fluid medium. Jabbari et al. [5,6] studied the mechanical and thermal stresses in a functionally graded hollow cylinder due to axisymmetric mechanical and thermal loads. Jabbari et al. [7] gave a general theoretical analysis of three-dimensional mechanical and thermal stresses for a short FGM hollow cylinder. Recently, Jabbari et al. [8] presented an analytical solution of one-dimensional mechanical and thermal stresses for a hollow cylinder made of functionally graded material. Zhao et al. [9] analyzed theoretically the transient thermo-mechanical behavior of a FGM solid cylinder under convective boundary condition. Dai et al. [10–13] studied the magnetothermoelastic interactions in hollow and solid

* Corresponding author at: Department of Engineering Mechanics, College of Mechanical and Vehicle Engineering, Hunan University, Changsha 410082, China.

E-mail address: hldai520@sina.com (H.-L. Dai).

0096-3003/$ - see front matter
doi:10.1016/j.amc.2011.06.030

Nomenclature

$\vec{U}, u$	displacement vector and radial displacement [m]
r	radial variable [m]
E_0, c_{ij}	elastic constants [N/m^2]
v	Poisson ratio
σ_r, σ_θ	components of stress [N/m^2]
$T(r)$	temperature distribution [K]
α_i, λ_i	thermal constants [1/K] and thermal modulus [N/m^2 K]
k	thermal conduction coefficient [W/mK]
h	ratio of the convective heat-transfer coefficient [W/K]
$\vec{J}$	electric current density vector
$\vec{h}$	perturbation of magnetic field vector
$\vec{e}$	perturbation of electric field vector
$\vec{H}$	magnetic intensity vector
μ_0	magnetic permeability [H/m]
f_z	Lorentz's force [kg/m^2 s^2]
a, b	inner and outer radii of the FGM hollow cylinder [m]
P_a, P_b	internal and external pressure of the FGM hollow cylinder [kg/m^2 s^2]

Non-dimensional quantities

R	$\frac{r-a}{b-a}$
T^*	$\frac{T(r)}{T_0}$
u^*	$\frac{u}{a}$
σ_r^*	$\frac{\sigma_r}{P_a}$
σ_θ^*	$\frac{\sigma_\theta}{P_a}$
h_z^*	$\frac{h_z}{H_z}$
$\sigma_r^\#$	$\frac{\sigma_r}{P_b}$
$\sigma_\theta^\#$	$\frac{\sigma_\theta}{P_b}$

cylindrical and spherical structures of functionally graded material subjected to mechanical loads. By means of the Fredholm integral equation, Li et al. [14] investigated radially polarized functionally graded piezoelectric hollow cylinders as sensors and actuators. Hosseini and Shahabian [15] presented the reliability analysis and safety evalution of dynamic stresses for Al–Al_2O_3 FG thick hollow cylinder subjected to sudden unloading as a mechanical shock loading.

This paper investigates magnetothermoelastic behaviors of a FGM hollow cylinder, placed in a uniform magnetic field, subjected to thermal and mechanical loads. A simple, tractable closed-form solution for the FGM hollow cylinder is presented. The emphasis of this research is laid on the effects of the inhomogeneous constants on magnetothermoelastic stresses and the perturbation of magnetic field vector. By means of practical example, it is possible for engineers to design a FGM cylindrical structure that can meet some special requirements by selecting proper values of $\beta_i(i = 1,2,3,4)$ and suitable loads.

2. Basic formulations of the problem

2.1. Derivation of equations

A long, FGM hollow cylinder with internal radius a and external radius b placed in a uniform magnetic field $\vec{H}(0,0,H_z)$ is shown in Fig. 1, the magnetothermoelastic stresses and perturbation of magnetic field vector distributions in the FGM hollow cylinders will be calculated. Letting the cylindrical coordinates of any representative point be (r,θ,z), the stiffness and thermal expansion coefficients are assumed to vary as $c_{ij} = c_{ij}^0 r^{\beta_1} (i = 1,2, \quad j = 1,2)$ and $\alpha_i(r) = \alpha_i^0 r^{\beta_2} (i = 1,2)$ through the wall thickness, respectively. Similar assumptions can be found in previous studies [2,16,17]. Here, c_{ij}^0 is the stiffness at the external surface $(r = b)$, α_i^0 are the thermal expansion coefficients at the external surface $(r = b)$ and $\beta_i(i = 1,2)$ is the inhomogeneous constant determined empirically. However, these values for $\beta_i(i = 1,2)$ do not necessarily represent a certain material. The range $\beta_i \in [-2,2]$ to be used in the present study covers the main part of the values of coordinate exponents encountered in the references cited earlier [2,3], and it is enough to demonstrate the effect of inhomogeneity on the