Applied Mathematics and Computation 218 (2012) 7792–7797

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

On reliability analysis of a two-dependent-unit series system with a standby unit

Serkan Eryilmaz [a,*], Fatih Tank [b]

[a] Atilim University, Department of Industrial Engineering, 06836 Incek, Ankara, Turkey
[b] Ankara University, Faculty of Science, Department of Statistics, 06100 Tandogan, Ankara, Turkey

ARTICLE INFO

Keywords:
Copula
Exchangeability
Mean time to failure
Positive dependence
Reliability

ABSTRACT

In this paper we study a series system with two active components and a single cold standby unit. The two simultaneously working components are assumed to be dependent and this dependence is modeled by a copula function. In particular, we obtain an explicit expression for the mean time to failure of the system in terms of the copula function and marginal lifetime distributions. We also provide illustrative numerical results for different copula functions and marginal lifetime distributions.

1. Introduction

Standby redundancy is an effective way to improve the reliability of an engineering system. In the study of system reliability under standby redundancy, the components are usually assumed to be independent. This assumption may not be valid in practice. Two components working in the common random environment may share the same load or may be subject to the same stress. In such a case, the lifetimes of the components are dependent. Most of the studies on systems with dependent components focus on the case when there is no standby component [7,8,1,9].

The notion of copulas has been found to be useful for modeling dependence in the context of system reliability [6,4,9,2]. In system reliability, copulas are used to create a multivariate lifetime distribution for modeling dependence among the components.

In this paper, we study two-unit series system with a single cold standby component whenever the two simultaneously working components are dependent through a given copula. In a series system with a single standby unit, the system consists of two serially connected working units and a single cold standby unit. The unit is said to be in the case of cold standby if it does not fail while in standby. When one of the working unit fails, then a cold standby unit is immediately put into operation. Some recent discussions on systems with standby component are in Jia and Wu [3], Wu and Wu [12], Wang and Zhang [11].

The paper is organized as follows. In Section 2, we define the lifetime of the system and compute its expected value, i.e. mean time to failure (MTTF). This section also includes bounds for the MTTF when the components are positively (negatively) quadrant dependent. Section 3 contains extensive numerical calculations for different copula functions and marginal lifetime distributions.

* Corresponding author.
E-mail addresses: seryilmaz@atilim.edu.tr, seryilmaz2002@yahoo.com (S. Eryilmaz).

0096-3003/$ - see front matter © 2012 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2012.01.046

2. Lifetime and MTTF of the system

Consider a two-unit series system with one cold standby component. Let X_i and Z represent respectively the lifetime of the ith active component ($i = 1, 2$) and the lifetime of the standby component. Then the lifetime of the system is represented as

$$T = \min(X_1, X_2) + \min(X_1^*, Z),$$

where X_1^* represents the residual lifetime of surviving active component after the first failure in the system, i.e.

$$X_1^* \overset{st}{=} (X_1 - \min(X_1, X_2) | X_1 > \min(X_1, X_2)).$$

The dependence between the components working in the common random environment is inevitable, and if the components are identical the common random environment makes them exchangeable dependent. In the present paper we model this dependence by the symmetric copula function C, i.e. $C(u, v) = C(v, u)$ for all $u, v \in [0, 1]$.

Let X_1 and X_2 be exchangeable dependent with an absolutely continuous joint distribution and this dependence is modeled by the copula function $C(u, v)$. That is, the joint cumulative distribution function (cdf) of X_1 and X_2 is given by

$$F(t_1, t_2) = P\{X_1 \leqslant t_1, X_2 \leqslant t_2\} = C(F(t_1), F(t_2)),$$

where $F(t) = P\{X_i \leqslant t\}, i = 1, 2$. In such a case $F(t_1, t_2) = F(t_2, t_1)$, i.e. X_1 and X_2 are exchangeable. Under these assumptions, the cdf of the random variable X_1^* is found to be

$$F^*(t) = P\{X_1^* \leqslant t\} = 2P\{X_1 \leqslant X_2 + t\} - 1 \tag{1}$$

for $t \geqslant 0$ (see Appendix). The latter cdf can be computed from the following equation using the joint distribution of X_1 and X_2 (see Appendix).

$$F^*(t) = 2\int_0^\infty \int_0^{y+t} c(F(x), F(y)) dF(x) dF(y) - 1, \tag{2}$$

where

$$c(u, v) = \frac{\partial^2}{\partial u \partial v} C(u, v).$$

Assume that the cold standby component has the same marginal distribution with X_is, i.e. $P\{Z \leqslant t\} = F(t)$. After the first failure in the system, the standby component is put into operation and it works together with the remaining component whose cdf is given by (1). Thus the joint distribution of X_1^* and Z are no longer exchangeable. They are again assumed to be dependent with the joint cdf

$$P\{X_1^* \leqslant t_1, Z \leqslant t_2\} = C(F^*(t_1), F(t_2))$$

for $t_1, t_2 \geqslant 0$.

The joint survival functions of (X_1, X_2) and (X_1^*, Z) are given respectively by

$$P\{X_1 > t_1, X_2 > t_2\} = 1 - F(t_1) - F(t_2) + C(F(t_1), F(t_2)) \tag{3}$$

and

$$P\{X_1^* > t_1, Z > t_2\} = 1 - F^*(t_1) - F(t_2) + C(F^*(t_1), F(t_2)). \tag{4}$$

Thus using (3) and (4), the survival functions of $\min(X_1, X_2)$ and $\min(X_1^*, Z)$ are found to be

$$P\{\min(X_1, X_2) > t\} = P\{X_1 > t, X_2 > t\} = 1 - 2F(t) + C(F(t), F(t))$$

and

$$P\{\min(X_1^*, Z) > t\} = 1 - F^*(t) - F(t) + C(F^*(t), F(t))$$

for $t \geqslant 0$.

Using these survival functions, the MTTF of the series system with a single cold standby unit is computed from

$$E(T) = \int_0^\infty [1 - 2F(t) + C(F(t), F(t))] dt + \int_0^\infty [1 - F^*(t) - F(t) + C(F^*(t), F(t))] dt. \tag{5}$$

There are various notions of dependence. A pair of random variables (X, Y) is said to be positively quadrant dependent (PQD) if

$$P\{X \leqslant t_1, Y \leqslant t_2\} \geqslant P\{X \leqslant t_1\} P\{Y \leqslant t_2\} \tag{6}$$