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Complex dynamics of duopoly game with heterogeneous players: A further analysis of the output model

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ABSTRACT

In this paper, an output duopoly game with heterogeneous players is analyzed in order to study the influence of players' different behavior on the dynamics of game. Two types of players are considered, which are bounded rationality expectation and naïve expectation. Player with naïve expectation chooses an output level based on the market price of previous period, while player with bounded rationality adjusts his output adaptively, following a bounded rationality adjustment process based on a local estimate of the marginal profit of previous period. The game model is also based on the assumption that demand and cost function are nonlinear. The existence of equilibrium points and its local stability of the output game are investigated. The complex dynamics, bifurcations and chaos are displayed by numerical experiment. Numerical methods also show that the long-run average profit achieved by player adopting naïve expectation is higher than that achieved by player using self adaptive adjustment measure, although players use similar production methods.

1. Introduction

An oligopoly has a market structure in which a trade is completely controlled by only a few number of firms in the market producing the same or homogeneous products (see [1,2]). When making a decision of production, firms must consider not only the information of the market but also the reactions of the competitors, so the dynamic of an oligopoly game may become more complex. Cournot [3] introduced the first well-known model which gives a mathematical description of competition in a duopolistic market. He assumed that each firm (player) chooses its production in order to maximize its profits according to output of its rival at previous period. Since Cournot, many models based on the Cournot model have been proposed to characterize the oligopoly game (see [2,4–15]).

Puu [16] may be the first man who found that Cournot adjustment process may take on complex dynamics, including period doubling bifurcations and chaos. Complex dynamics with three oligopolists was discussed by Puu [12]. Ahmed and Agiza [7] generalized Puu's work to n-competitors. As pointed out by Ahmed et al. [8], there are some unrealistic assumptions of Puu [1,2] that each firm knows the production of its rival, and knows the market demand function. A more realistic approach is to assume player with bounded rationality, that is, each firm adjust its production based on a local estimate of the marginal profit. It has been well-documented that economic system has nonlinear interaction among its components (see [4,5,14,17–23]).

Expectations also play an important role in modeling of economic phenomena. A producer can choose his expectations rules from many available techniques to adjust his production outputs. Cournot [3] investigated the case that each firm is

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provided with naïve expectations in duopoly, that is, in every step each firm assumes that the output of his competitor at this period is the same as that of previous period. Bounded rationality expectation has been considered by some scholars [4,5,8,9,14]. In all of these documents the players are assumed to have homogeneous expectations rules for computing their expected outputs. However, the types of players may be different in real business world. Economic models with heterogeneous players have been proposed in Agiza and Elsadany [2,6], Borck and Hommes [24], Den-Haan [25], Ding et al. [26], Dubiel-Teleszynski [27], Léonard and Nishmura [11], Onozaki et al. [28] and Zhang et al. [15]. Agiza and Elsadany [6] applied the technique of Onazaki et al. [28] to study the dynamics of Cournot duopoly model which contains two heterogeneous players: one is boundedly rational and the other is naïve. They [2] further considered the duopoly model with two heterogeneous players: one has bounded rationality and the other has adaptive expectations. Ding et al. [26] studied the dynamics of a two-team Cournot game played by one team consisting of two firms with bounded rationality and the other team consisting of one firm with naïve expectation. Dubiel-Teleszynski [27] discussed dynamics in a heterogeneous duopoly game with adjusting players and diseconomies of scale, and found that industries facing diseconomies of scale are found to be less stable than those with constant marginal costs. Zhang et al. [15] extended the work of [6] by modifying the linear cost function. In the works of [2,6,15], they assumed that naïve player adjust their production in the light of rival's output of previous period, and the demand function is linear. In this paper, we suppose that naïve player do not know his rival's output, and adjust his production in term of production's market price of previous period, which has been well-documented, but has not been used in the Cournot duopoly game. In order to modeling the nonlinear interaction of the variables, we study the case of the model with nonlinear demand function and nonlinear cost function. As showed by Du and Huang [29] and Du et al. [30], players should pay attention to the performances of economic systems in various period states and chaotic states so as to cope with complexity. In this paper, we use long-run average profit to measure the performance of player with different expectations, which is not considered by [2,6,11,15,25–28].

The remainder of this paper is organized as follows. Section 2 introduces and describes an output duopoly game with heterogeneous players. In Section 3, dynamics of the output game is analyzed. Existence, local stability and bifurcation of equilibrium points are given. Section 4 shows complex dynamics of the output game by computing the largest Lyapunov exponents, and sensitivity to initial conditions. The performances of players with different expectations are also compared in this section. The final section concludes the paper.

2. The model

We consider a duopoly competition between two firms (players) producing the homogeneous product. Then one firm is labeled by $i = 1$, and the other $i = 2$. The strategy space is the choice of the output, and the decision-making takes place in the discrete time periods $t = 0,1,2,\ldots$: $q_i(t)$ represent the output of the ith firm during period t. The price p, a nonlinear inverse demand function, is determined by:

$$p = f(Q) = a - b\sqrt{Q}, \tag{1}$$

where $Q(t) = q_1(t) + q_2(t)$ is the total supply and $a, b > 0$. We assume that the production cost function has the nonlinear form.

$$C_i(q_i) = c_i + dq_i + eq_i^2, \quad i = 1,2, \tag{2}$$

where the producers use similar production methods, c_i is positive parameter representing fixed cost, which may be different for production scale's difference of each firm. The cost function $C_i(q_i)$, climbing with the increase of the product output, is convex usually, so its first derivative $C_i'(q_i)$ and second derivative $C_i''(q_i)$ are positive. We can assume that the parameters d, e are positive [30]. In order to make the two-players' game on the rails, the marginal cost of the ith firm ($i = 1,2$) is less than the highest price of the homogeneous product in the market. Therefore, $d + 2eq_i < a$, $i = 1,2$. That is $q_i < (a - d)/2e$. At last, the profit of the ith firm at the period t is:

$$\Pi_i(q_1, q_2) = q_i(a - b\sqrt{Q}) - (c_i + dq_i + eq_i^2), \quad i = 1,2. \tag{3}$$

Then the marginal profit of the ith firm at the period t is:

$$\frac{\partial \Pi_i(q_1, q_2)}{\partial q_i} = a - b\sqrt{Q} - \frac{bq_i(t)}{2\sqrt{Q}} - d - 2eq_i, \quad i = 1,2. \tag{4}$$

In order to get the maximum profit, every firm carries out the output decision-making. In this work, we consider two firm with different expectation. Suppose that the ith = 1st firm with bounded rationality adjusts its production based on a local estimate of the marginal profit $\partial\Pi_1/\partial q_1$ (see [8]). That is to say, if i = 1st firm thinks the marginal profit at the period t is positive, it will increase the output of at the period $t + 1$; whereas if the marginal profit is negative, it will decrease the output. Then the dynamic adjustment mechanism of the i = 1st firm has the form:

$$q_1(t+1) = q_1(t) + \alpha q_1(t)\frac{\partial \Pi_1(q_1, q_2)}{\partial q_1}, \tag{5}$$

where α is positive parameter representing the speed of adjustment of the i = 1st firm. We take (4) into (5), then we get the following form: