Performance Evaluation 73 (2014) 91–109

Contents lists available at ScienceDirect

Performance Evaluation

journal homepage: www.elsevier.com/locate/peva

Lower-bound-constrained runs in weighted timed automata

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ARTICLE INFO

Article history:
Available online 1 December 2013

Keywords:
Weighted timed automata
Energy constraints
Time-bounded verification

ABSTRACT

We investigate a number of problems related to infinite runs of weighted timed automata (with a single weight variable), subject to lower-bound constraints on the accumulated weight. Closing an open problem from Bouyer et al. (2008), we show that the *existence* of an infinite lower-bound-constrained run is—for us somewhat unexpectedly—undecidable for weighted timed automata with four or more clocks.

This undecidability result assumes a fixed and known initial credit. We show that the related problem of *existence of an initial credit* for which there exists a feasible run is decidable in PSPACE. We also investigate the variant of these problems where only bounded-duration runs are considered, showing that this restriction makes our original problem decidable in NEXPTIME. We prove that the *universal* versions of all those problems (i.e, checking that *all* the considered runs satisfy the lower-bound constraint) are decidable in PSPACE.

Finally, we extend this study to multi-weighted timed automata: the existence of a feasible run becomes undecidable even for bounded duration, but the existence of initial credits remains decidable (in PSPACE).

1. Introduction

Weighted (or *priced*) *timed automata* [1–3] have emerged as a useful formalism for formulating a wide range of resource-allocation and optimization problems [4,5], with applications in areas such as embedded systems [6]. In [7], a new class of resource-allocation problems was introduced, namely that of constructing infinite schedules subject to boundary constraints on the accumulation of resources.

More specifically, we proposed weighted timed automata with positive as well as negative weight-rates in locations, allowing for the modeling of systems where resources (e.g. energy) are not only consumed but also possibly produced. As a basic example, consider the two-clock weighted timed automaton $\mathcal{A}$ in Fig. 1 with infinite behaviors repeatedly delaying in ℓ_0, ℓ_1, ℓ_2 and ℓ_3 for a total duration of two time units, with one time unit spent in ℓ_0 and ℓ_3 and one time unit spend in ℓ_1 and ℓ_2 (we silently assume an invariant on all locations, imposing that clock y has to always remain below 2). The values ($+2$, $+3$ and $+4$) in the four locations indicate the rate by which energy is produced (or consumed, when negative), and the values (-2 and -3) on the edges indicate instantaneous updates to the energy level (there is only one weight variable in this example). Clearly, the energy remaining after a given iteration will depend not only on the initial energy but also highly on the distribution of the two time units over the four locations.

In this paper we consider a number of problems related to infinite runs subject to lower-bound constraints on the accumulated weights (e.g. infinite runs where the energy level never goes below zero). In the absence of an upper bound and if there is only one weight variable, it suffices to consider runs along which the accumulated weight is maximized. Fig. 2

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http://dx.doi.org/10.1016/j.peva.2013.11.002

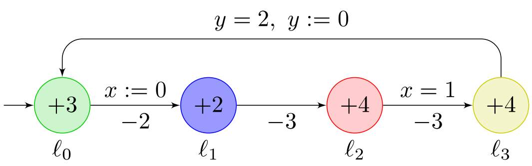

Fig. 1. A 2-clock weighted timed automaton $\mathcal{A}$ (with implicit global invariant $y \leq 2$).

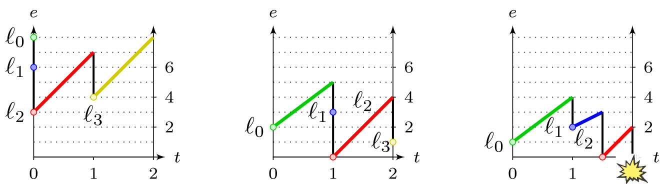

Fig. 2. Three possible behaviors (representing the evolution of the location and energy level with time) in $\mathcal{A}$ (with initial credits 8, 2 and 1, resp.).

illustrates three such energy-maximizing behaviors of $\mathcal{A}$. For initial energy 8, the maximum energy left after one iteration is 8, thus providing an infinite lower-bound schedule. In contrast, an initial energy level of 1 does not even permit a single iteration (let alone an infinite schedule), and an initial energy level 2 leaves at maximum 1, for which we already know that no infinite lower-bound schedule exists. In this simple (non-branching) example, it can be shown that 1.5 is the least initial credit for which it is possible to come back to ℓ_0, and 5 is the minimal initial credit that allows an infinite-duration run.

For weighted timed automata with a single clock and a single weight variable, the existence of a lower-bound constrained infinite run has been shown decidable in polynomial time [7] with the restriction that no discrete updates of the accumulated weight occur on transitions. In [8], it is shown that the problem remains decidable if this restriction is lifted and even if the accumulated weight grows not only linearly but also exponentially. In contrast, the existence of *interval-constrained* infinite runs—where a simple energy-maximizing strategy does not suffice—has recently been proven undecidable for weighted timed automata with varying numbers of clocks and weight variables: e.g. two clocks and two weight variables [9], one clock and two weight variables [10], and two clocks and one weight variable [11]. Also, the interval-constrained problem is undecidable for weighted timed automata with one clock and one weight variable in the *game* setting [7].

Still, the general problem of *existence* of infinite lower-bounded runs for weighted timed automata has remained unsettled since [7]. In this paper we close this open problem showing that it is undecidable for weighted timed automata with four or more clocks and one weight variable. Given that this problem looks rather simple (since there is only one weight variable, it suffices to consider energy-maximizing runs), we find this result quite surprising and somewhat disappointing. Thus, we consider a number of related problems for which we show decidability and settle complexity. In particular, the undecidability result assumes a fixed and known initial energy level. We show that the related problem of *existence of an initial energy level* allowing an infinite lower-bound constrained run is decidable in PSPACE in the one-weight case. We also investigate the variant of these problems, where the lower-bound constraint is only imposed for a limited duration: for instance, for the weighted timed automaton in Fig. 1 and initial energy level of 4, we may want to settle the existence of a run along which the energy level remains non-negative during the first 4.7 time units, say. Note that the time-bounded paradigm has recently emerged as a pertinent restriction option for the verification of real-time systems [12] (in quite the same way as bounded model checking has been used for untimed systems [13]). We show that this restriction makes our original problem decidable and NEXPTIME-complete (assuming only one weight variable). Our result has to be compared with rectangular hybrid automata, for which time-bounded reachability has recently been shown decidable in EXPSPACE (no matching lower bound is provided, though), under the hypothesis that all rates are non-negative (if rates can be negative, the problem is undecidable) [14]. Our model of weighted timed automata is a special case of rectangular hybrid automata, in which all variables are clocks (rate 1) and one variable can have non-negative as well as negative rates. Therefore none of the two decidability results implies the other. We refer to Table 1 for a summary of the aforementioned results.

We also extend this study to multi-weighted timed automata, showing that the above decidability result for the existence of a time-bounded constrained run does not carry over to that multi-dimensional setting (our undecidability proof uses ten weight variables, but only one clock). Still, the existence of an initial credit in the time-bounded setting is proven to remain decidable.

Finally, we also consider the *universal* versions of all the above problems (i.e., checking that *all* the considered runs satisfy the lower-bound constraint), and prove that they all are decidable in PSPACE.

2. Definitions

2.1. Basic definitions

We write $\mathbb{R}_{\geq 0}$, $\mathbb{Q}_{\geq 0}$ and $\mathbb{N}$ respectively for the set of non-negative reals, rationals and integers. We assume that X is a finite set of variables called *clocks*. A valuation v of the clocks is a mapping $X \to \mathbb{R}_{\geq 0}$. If v is a valuation and $t \in \mathbb{R}_{\geq 0}$,