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Observer synthesis method for Lipschitz nonlinear discrete-time systems with time-delay: An LMI approach

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ABSTRACT

In this paper, we address the problem of observer design for a class of Lipschitz nonlinear discrete-time systems with time-delay. The main contribution lies in the use of a new structure of the proposed observer with a novel Lyapunov–Krasovskii functional. Thanks to these designs, new nonrestrictive synthesis conditions, expressed in terms of linear matrix inequalities (LMIs), are obtained. Indeed, the obtained LMIs contain more degree of freedom than those established by the approaches available in the literature which consider a simple Luenberger observer with a simple Lyapunov function for the stability analysis. An extension of the presented result to $\mathcal{H}_\infty$ performance analysis is given in order to take into account the noise (if it exists) affecting the considered system.

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1. Introduction

The observer design problem for nonlinear time-delay systems becomes more and more a subject of tremendous research activities over the last decades [1–8]. Indeed, time-delay is frequently encountered in various practical systems, such as chemical engineering systems, neural networks and population dynamic model. An overview of some recent advances and open research problems is summarized in [9]. One of the recent application of time-delay is the synchronization and information recovery in chaotic communication systems [10,11]. In fact, the time-delay is added in a suitable way to the chaotic system in the goal to increase the complexity of the chaotic behavior and then to enhance the security of communication systems. On the other hand, contrary to nonlinear continuous-time systems, little attention has been paid toward the discrete-time case [12,13]. In [12], the authors investigated the problem of robust $\mathcal{H}_\infty$ observer design for a class of Lipschitz time-delay systems with uncertain parameters in the discrete-time case. Their method show the stability of the state of the system and the estimation error simultaneously.

This paper deals with observer design for a class of Lipschitz nonlinear discrete-time systems with time-delay. The main result lies, first, in the use of a new structure of the proposed observer inspired from [14], and on the other hand, in the use of a novel Lyapunov–Krasovskii functional which depends on the nonlinear term of the considered system. New synthesis conditions are obtained. These conditions, expressed in terms of LMIs, contain more degree of freedom than those proposed by the approaches available in the literature. Indeed, these last use a simple Luenberger observer and a classical Lyapunov–Krasovskii functional, which can be derived from the general forms proposed in this paper by neglecting some matrices. An extension of the presented result to $\mathcal{H}_\infty$ performance analysis is given in the goal to take into account the noise which affects the considered system. A more general LMI is established.

The rest of this paper is arranged as follows. In Section 2, we introduce the class of systems under consideration, the proposed observer and new sufficient synthesis conditions (LMIs conditions). In Section 3, we give an extension of the presented

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result to $\mathcal{H}_\infty$ performance analysis. Section 4 concludes the proposed work. Finally, we consider an appendix to detail some concepts.

Notations: The following notations will be used throughout this paper.

- $\|\cdot\|$ is the usual Euclidean norm;
- $(\star)$ is used for the blocks induced by symmetry;
- A^T represents the transposed matrix of A;
- I_r represents the identity matrix of dimension r;
- for a square matrix S, $S>0(S<0)$ means that this matrix is positive definite (negative definite);
- $z_t(k)$ represents the vector $x(k-t)$ for all z;
- The notation $\|x\|_{\ell_2^s} = \left(\sum_{k=0}^{\infty}\|x(k)\|^2\right)^{\frac{1}{2}}$ is the ℓ_2 norm of the vector $x \in \mathbb{R}^s$. The set ℓ_2^s is defined by

$$\ell_2^s = \left\{x \in \mathbb{R}^s : \|x\|_{\ell_2^s} < +\infty\right\}.$$

In the next section, we introduce the class of systems to be studied, the proposed observer and the synthesis condition ensuring the asymptotic convergence of the observer.

2. Problem formulation and observer synthesis conditions

In this section, we introduce the class of nonlinear systems to be studied, the proposed state observer and the observer synthesis conditions.

2.1. Problem formulation

Consider the class of systems described in a detailed form by the following equations:

$$x(k+1) = Ax(k) + A_d x_d(k) + Bf(Hx(k), H_d x_d(k)), \tag{1a}$$
$$y(k) = Cx(k), \tag{1b}$$
$$x(k) = x^0(k), \quad \text{for} \quad k = -d, \ldots, 0, \tag{1c}$$

where the constant matrices A, A_d, B, C, H and H_d are of appropriate dimensions and d is the time-delay

The function $f : \mathbb{R}^{s_1} \times \mathbb{R}^{s_2} \to \mathbb{R}^q$ satisfies the Lipschitz condition with Lipschitz constant γ_f, i.e. :

$$\|f(z_1, z_2) - f(\hat{z}_1, \hat{z}_2)\| \leqslant \gamma_f \left\| \begin{bmatrix} z_1 - \hat{z}_1 \\ z_2 - \hat{z}_2 \end{bmatrix} \right\|, \quad \forall\, z_1, z_2, \hat{z}_1, \hat{z}_2. \tag{2}$$

Now, consider the following new structure of the proposed observer defined by the Eqs. (3):

$$\hat{x}(k+1) = A\hat{x}(k) + A_d\hat{x}_d(k) + Bf(v(k), w(k)) + L(y(k) - C\hat{x}(k)) + L_d(y_d(k) - C\hat{x}_d(k)), \tag{3a}$$
$$v(k) = H\hat{x}(k) + K^1(y(k) - C\hat{x}(k)) + K_d^1(y_d(k) - C\hat{x}_d(k)), \tag{3b}$$
$$w(k) = H_d\hat{x}_d(k) + K^2(y(k) - C\hat{x}(k)) + K_d^2(y_d(k) - C\hat{x}_d(k)). \tag{3c}$$

The dynamics of the estimation error is:

$$\varepsilon(k+1) = (A - LC)\varepsilon(k) + (A_d - L_d C)\varepsilon_d(k) + B\delta f_k, \tag{4}$$

with

$$\delta f_k = f(Hx(k), H_d x_d(k)) - f(v(k), w(k)).$$

From (2), we have

$$\|\delta f_k\| \leqslant \gamma_f \left\| \begin{bmatrix} (H - K^1 C)\varepsilon(k) - K_d^1 C\varepsilon_d(k) \\ \left(H_d - K_d^2 C\right)\varepsilon_d(k) - K^2 C\varepsilon(k) \end{bmatrix} \right\|. \tag{5}$$

The next subsection is devoted to the observer synthesis method that provides a sufficient condition ensuring the asymptotic convergence of the estimation error towards zero.

2.2. Observer synthesis conditions

The synthesis conditions, expressed in term of LMIs, are given in the following theorem:

Theorem 2.1. *The estimation error is asymptotically stable if there exist a scalar $\alpha > 0$ and matrices $P = P^T > 0, Q = Q^T > 0$, $S = S^T > 0, X = X^T > 0, Y = Y^T > 0, R, R_d, \overline{K}^1, \overline{K}^2, \overline{K}_d^1$ and $\overline{K}_d^2$ of appropriate dimensions such that the following LMIs are feasible:*

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