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On the numerical solution of Korteweg–de Vries equation by the iterative splitting method

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Dedicated to Professor H. M. Srivastava on the Occasion of his Seventieth Birth Anniversary

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ABSTRACT

In this paper, we apply the method of iterative operator splitting on the Korteweg–de Vries (KdV) equation. The method is based on first, splitting the complex problem into simpler sub-problems. Then each sub-equation is combined with iterative schemes and solved with suitable integrators. Von Neumann analysis is performed to achieve stability criteria for the proposed method applied to the KdV equation. The numerical results obtained by iterative splitting method for various initial conditions are compared with the exact solutions. It is seen that they are in a good agreement with each other.

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1. Introduction

Nonlinear wave equations are widely used to describe complex phenomena in various sciences such as fundamental particle physics, plasma and fluid dynamics, statistical mechanics, protein dynamics, condensed matter, biophysics, nonlinear optics, quantum field theory, see [14,3,1,6]. The wide applicability of these equations is the main reason why they have attracted so much attention from many mathematicians. However, they are usually very difficult to solve, either numerically or analytically.

During the past four decades, both mathematicians and physicists have devoted considerable effort to the study of exact and numerical solutions of the nonlinear partial differential equations corresponding to the nonlinear problems. Many powerful methods have been presented, for instance, Darboux transformation method [9], Adomians decomposition method [15,11], He's perturbation method [16], Operator splitting method [10], Iterative splitting method [7].

In this paper, we consider the nonlinear Korteweg–de Vries (KdV) equation

$$u_t + 6uu_x + u_{xxx} = 0, \tag{1}$$

which was found to admit soliton solutions and be able to model the propagation of solitary wave on water surface. Its phenomena was first discovered by Russell in 1834 [13] and Korteweg–de Vries formulated the mathematical model equation to provide explanation of the phenomena. In [11,16,2], KdV equation has been solved with Adomain's decomposition (ADM), He's perturbation method (HPM) and a particle method (based on diffusion-velocity method) analytically and numerically. Here, we use iterative operator splitting method to study on the nonlinear KdV equation.

The iterative splitting is a recent popular technique which is based on first splitting the complex problem into simpler differential equations. Then each sub-equation is combined with the iterative schemes, each of which is efficiently solved with suitable integrators [5,4,8,7].

Furthermore, this study explicitly derives the stability criteria for iterative splitting method using Fourier analysis, which based on KdV equation [12].

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The structure of the paper is as follows: In Section 2, outline of the iterative splitting method is given. Stability analysis of the method which based on KdV equation is derived in Section 3. In Section 4, applications of the method on KdV equation is done. Finally, we have numerical results and conclusion part.

2. Outline of the method

Consider the abstract Cauchy problem

$$u'(t) = (A + B)u(t), \quad t \in [0, T], \tag{2}$$

$$u(0) = u_0, \tag{3}$$

where A and B are bounded linear operators and u_0 is initial condition. For such problem, the exact solution can be given as

$$u(t) = \exp((A + B))u_0, \quad t \in [0, T]. \tag{4}$$

The method is based on iteration by fixing the splitting discretization step size Δt on time interval $[t^n, t^{n+1}]$. The following algorithms are then solved consecutively for $i = 1, 3, \ldots, 2m + 1$.

$$u_i'(t) = Au_i(t) + Bu_{i-1}(t) \text{ with } u_i(t^n) = u^n, \tag{5}$$

$$u_{i+1}'(t) = Au_i(t) + Bu_{i+1}(t) \text{ with } u_{i+1}(t^n) = u^n, \tag{6}$$

where u^n is the known split approximation at time level $t = t^n$ and $u_0 \equiv 0$ is the initial guess. The split approximation at the time-level $t = t^{n+1}$ is defined as $u^{n+1} = u_{2m+2}(t^n)$.

3. Stability analysis of iterative splitting method on KdV equation via von Neumann

In this section, we will investigate the stability analysis of iterative splitting method for KdV equation via von Neumann approach. Consider again the KdV equation of the form

$$u_t + 6uu_x + u_{xxx} = 0. \tag{7}$$

Firstly, split Eq. (7) into two parts

$$u_t = -u_{xxx} \text{ and } u_t = -6uu_x \tag{8}$$

and apply iterative splitting schemes, then have the following algorithms:

$$u_i' = -(u_i)_{xxx} + 6u_{i-1}(u_{i-1})_x, \tag{9}$$

$$u_{i+1}' = -(u_i)_{xxx} + 6u_i(u_{i+1})_x, \tag{10}$$

where $i = 1, 3, \ldots, 2m + 1$.

Note that, in this approach, it is not necessary to specify a spatial discretization technique.

Rearrangement of algorithms (19) and (20) with a linearization about steady state $6u_{i-1} = k_1$, $6u_i = k_2$ yields

$$u_i' = L_1 u_i + k_1 L_2 u_{i-1}, \tag{11}$$

$$u_{i+1}' = L_1 u_i + k_2 L_2 u_{i+1}, \tag{12}$$

where $L_1 = -\frac{\partial^3}{\partial x^3}$, $L_2 = -\frac{\partial}{\partial x}$ and $i = 1, 3, \ldots, 2m + 1$.

Secondly, combine algorithms (11) and (12) with the second order midpoint rule then have

$$\begin{pmatrix} u_i^{n+1} \\ u_{i+1}^{n+1} \end{pmatrix} = \begin{pmatrix} u_i^n \\ u_{i+1}^n \end{pmatrix} + \Delta t \begin{pmatrix} L_1 \frac{u_i^n + u_i^{n+1}}{2} + k_1 L_1 \frac{u_{i-1}^n + u_{i-1}^{n+1}}{2} \\ L_1 \frac{u_i^n + u_i^{n+1}}{2} + k_2 L_2 \frac{u_{i+1}^n + u_{i+1}^{n+1}}{2} \end{pmatrix}, \tag{13}$$

where Δt is the time step on $[t^n, t^{n+1}]$ interval.

Finally, Eq. (13) can be put in the following matrix form

$$\begin{pmatrix} \tilde{u}_i^{n+1} \\ \tilde{u}_{i+1}^{n+1} \end{pmatrix} = \begin{pmatrix} \frac{1 + \frac{\Delta t}{2} w^3 i}{1 - \frac{\Delta t}{2} w^3 i} & 0 \\ \frac{\frac{\Delta t}{2} w^3 i \left(1 + \frac{\Delta t}{2} w^3 i\right)}{\left(1 - \frac{\Delta t}{2} w^3 i\right)\left(1 + \frac{\Delta t}{2} k_2 w i\right)} + \frac{\frac{\Delta t}{2} w^3 i}{1 + \frac{\Delta t}{2} k_2 w i} & \frac{1 - \frac{\Delta t}{2} k_2 w i}{1 + \frac{\Delta t}{2} k_2 w i} \end{pmatrix} \begin{pmatrix} \tilde{u}_i^n \\ \tilde{u}_{i+1}^n \end{pmatrix} = \widetilde{A} \begin{pmatrix} \tilde{u}_i^n \\ \tilde{u}_{i+1}^n \end{pmatrix}, \tag{14}$$

by taking the fourier transform according to the formula

$$\hat{u}(w) = \frac{1}{\sqrt{2\pi}} \int_{\mathbf{R}} e^{-iwx} u(x) dx. \tag{15}$$

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