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Some families of generating functions for the extended Srivastava polynomials

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Dedicated to Professor H. M. Srivastava on the Occasion of his Seventieth Birth Anniversary

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ABSTRACT

Almost four decades ago, H.M. Srivastava considered a general family of univariate polynomials, the Srivastava polynomials, and initiated a systematic investigation for this family [10]. In 2001, B. González, J. Matera and H.M. Srivastava extended the Srivastava polynomials by inserting one more parameter [4]. In this study we obtain a family of linear generating functions for these extended polynomials. Some illustrative results including Jacobi, Laguerre and Bessel polynomials are also presented. Furthermore, mixed multilateral and multilinear generating functions are derived for these polynomials.

1. Introduction

Over four decades ago, Srivastava [10] introduced the Srivastava polynomials,

$$S_n^N(z) = \sum_{k=0}^{[\frac{n}{N}]} \frac{(-n)_{Nk}}{k!} A_{n,k} z^k \quad (n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}; N \in \mathbb{N}),$$

where $\mathbb{N}$ is the set of positive integers, $\{A_{n,k}\}_{n,k=0}^{\infty}$ is a bounded double sequence of real or complex numbers, $[a]$ denotes the greatest integer in $a \in \mathbb{R}$, and $(\lambda)_v$, $(\lambda)_0 \equiv 1$, denotes the Pochhammer symbol defined by

$$(\lambda)_v := \frac{\Gamma(\lambda + v)}{\Gamma(\lambda)}$$

in terms of familiar Gamma functions. In [4], González et al. extended the Srivastava polynomials $S_n^N(z)$ as follows:

$$S_{n,m}^N(z) = \sum_{k=0}^{[\frac{n}{N}]} \frac{(-n)_{Nk}}{k!} A_{n+m,k} z^k \quad (m, n \in \mathbb{N}_0; N \in \mathbb{N}), \tag{1}$$

and investigated their properties extensively. We call these polynomials as extended Srivastava polynomials since $S_{n,0}^N(z) = S_n^N(z)$. It has been shown in [4] that the extended Srivastava polynomials include many well known polynomials such as Laguerre, Jacobi and Bessel polynomials under the special choices, which we recall them below:

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Remark 1.1. Choosing $A_{m,n} = (-\alpha - m)_n (m, n \in \mathbb{N}_0)$ in (1), then one has

$$S^1_{n,m}\left(\frac{-1}{z}\right) = \frac{n!}{(-z)^n} L_n^{(\alpha+m)}(z),$$

where $L_n^{(\alpha)}(z)$ is the classical Laguerre polynomial given by

$$L_n^{(\alpha)}(z) = \frac{(-z)^n}{n!}\, {}_2F_1\left(-n, -\alpha - n; -; \frac{-1}{z}\right).$$

Remark 1.2. Setting

$$A_{m,n} = \frac{(\alpha+\beta+1)_{2m}(-\beta-m)_n}{(\alpha+\beta+1)_m(-\alpha-\beta-2m)_n} \quad (m, n \in \mathbb{N}_0)$$

in Eq. (1), then it is known that [4, pp. 146, eq. 38],

$$S^1_{n,m}\left(\frac{2}{1+z}\right) = n!(\alpha+\beta+m+n+1)_m \left(\frac{2}{1+z}\right)^n P_n^{(\alpha+m,\beta+m)}(z),$$

where $P_n^{(\alpha,\beta)}(z)$ are the classical Jacobi polynomials.

Remark 1.3. If we set $A_{m,n} = (\alpha + m - 1)_n (m, n \in \mathbb{N}_0)$ in Eq. (1), then we can write

$$S^1_{n,m}\left(\frac{-z}{\beta}\right) = y_n(z, \alpha + m, \beta), \quad (\beta \neq 0),$$

where $y_n(z,\alpha,\beta)$ is the classical Bessel polynomial [5] given by

$$y_n(z,\alpha,\beta) = {}_2F_0\left(-n, \alpha + n - 1; -; \frac{-z}{\beta}\right).$$

Recently, different variants of the polynomials $S_n^N(z)$ have been investigated in [6] and [7]. The bivariate version of the extended Srivastava polynomials, which includes many well known polynomials such as Lagrange-Hermite polynomials, Lagrange polynomials and Hermite–Kamp é de Feriét polynomials, has been introduced in [1] and further studied in [9]. Very recently, the three variable version of the polynomials $S_{n,m}^N(z)$ has been investigated in [12].

In the present paper we obtain a family of linear generating functions for extended Srivastava polynomials by using certain hypergeometric transformation. Some special cases of the main result including Jacobi, Laguerre and Bessel polynomials are also exhibited. Furthermore, we derive several families of mixed multilinear and multilateral generating functions for these polynomials.

2. Main results

In this section, by using the hypergeometric transformation,

$$ {}_2F_1(a, b; 2a; x) = \left(1 - \frac{x}{2}\right)^{-b} {}_2F_1\left(\frac{b}{2}, \frac{b+1}{2}; a + \frac{1}{2}; \left(\frac{x}{2-x}\right)^2\right), \tag{2}$$

provided that each member of Eq. (2) exists (see [2][pp. 127]), we obtain families of linear generating functions for the polynomials $S_n^{m,N}(z)$ defined by Eq. (1). The main result of the paper is the following theorem.

Theorem 2.1. *Let the polynomials $S_n^{m,N}(z)$ be defined by Eq. (1). Then, for a suitably bounded sequence $\{f(n)\}_{n\in\mathbb{N}_0}$, the following family of linear generating relations holds true*

$$\sum_{m,n=0}^{\infty} f(m+n) \frac{(\lambda)_m}{(2\lambda)_m} S_n^{m,N}(z) \frac{x^n y^m}{n!m!} = \sum_{m,n,k=0}^{\infty} \frac{f(n+2m+Nk)A_{n+2m+Nk,k}}{(\lambda+\frac{1}{2})_m} \frac{y^{2m}}{2^{4m}m!} \frac{(2x+y)^n}{2^n n!} \frac{\left[(-x)^N z\right]^k}{k!}, \tag{3}$$

provided that each member of the assertion Eq. (3) exists.

Proof. For convenience, let $\Psi(x,y,z)$ denote the first member of the assertion Eq. (3). Then, upon substituting for the polynomials $S_n^{m,N}(z)$ from the definition Eq. (1) into the left-hand side of Eq. (3), we obtain