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Construction a new generating function of Bernstein type polynomials

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Dedicated to Professor H. M. Srivastava on the Occasion of his Seventieth Birth Anniversary

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ABSTRACT

Main purpose of this paper is to reconstruct generating function of the Bernstein type polynomials. Some properties of this generating functions are given. By applying this generating function, not only derivative of these polynomials but also recurrence relations of these polynomials are found. Interpolation function of these polynomials is also constructed by Mellin transformation. This function interpolates these polynomials at negative integers which are given explicitly. Moreover, relations between these polynomials, the Stirling numbers of the second kind and Bernoulli polynomials of higher order are given. Furthermore some remarks associated with the Bezier curves are given.

1. Introduction, definitions and preliminaries

The Bernstein basis polynomials, recently, have been defined by many different ways, for examples in q-series, by complex function and many algorithms. These polynomials are used not only approximations of functions in various, but also in the other fields such as smoothing in statistics, in numerical analysis, in the solution of the differential equations, constructing Bezier curves and in computer aided geometric design cf. ([2,8,3,5,7,11,1]), and see also the references cited in each of these earlier works.

By the same motivation of Ozden' [6] paper, which is related to the unification of the Bernoulli, Euler and Genocchi polynomials, we, in this paper, construct a generating function of the Bernstein basis polynomials which unify generating function in [11,1].

2. Construction generating functions of the Bernstein basis polynomials

In this section we unify generating function of the Bernstein basis polynomials. We define

$$\mathcal{F}(t,b,s:x) = \frac{2^b x^{bs} \left(\frac{t}{2}\right)^{bs} e^{t(1-x)}}{(bs)!}$$

where $b,s \in \mathbb{Z}^+ = \{1,2,3,\cdots\}$, $t \in \mathbb{C}$ and $x \in [0,1]$. This function is generating function of the polynomials $\mathfrak{S}_n(bs,x)$:

$$\mathcal{F}(t,b,s:x) = \sum_{n=0}^{\infty} \mathfrak{S}_n(bs,x) \frac{t^n}{n!}. \tag{2.1}$$

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Remark 1. If we set $s = 1$ in (2.1), we obtain

$$\frac{(xt)^b e^{t(1-x)}}{b!} = \sum_{n=0}^{\infty} B_b^n(x)\frac{t^n}{n!}, \mathrm{cf.}([11,1])$$

and $\mathfrak{S}_n(b,x) = B_b^n(x)$, which denotes the Bernstein basis polynomials cf. ([2,3,5,8,11,1]).

By using the Taylor series for $e^{t(1-x)}$ in (2.1), we arrive at the following theorem:

Theorem 1. *Let $x \in [0,1]$. Let b, n and s be nonnegative integers. If $n \geqslant bs$, then*

$$\mathfrak{S}_n(bs,x) = \binom{n}{bs}\frac{x^{bs}(1-x)^{n-bs}}{2^{b(s-1)}}.$$

Remark 1. Setting $s = 1$ in Theorem 1, one can see that the polynomials

$$\mathfrak{S}_n(b,x) = B_b^n(x) = \binom{n}{b}x^b(1-x)^{n-b},$$

which give us the Bernstein basis polynomials cf. ([2,8,3,5,9,7,11,1]). Consequently, the polynomials $\mathfrak{S}_n(bs,x)$ are unification of the Bernstein basis polynomials.

By using Theorem 1, we easily obtain the following results.

Corollary 1. *Let b, n and s be nonnegative integers with $n \geqslant bs$. Then*

$$2\binom{n}{bs}\mathfrak{S}_{n+1}(bs+1,x) = x\binom{n+1}{bs+1}\mathfrak{S}_n(bs,x).$$

Setting

$$\mathfrak{g}_n(bs,x) = 2^{b(s-1)}\mathfrak{S}_n(bs,x).$$

For $bs = j$, we have partitions of unity as follows:

$$\sum_{j=0}^{n}\mathfrak{g}_n(j,x) = 1.$$

Let f be a continuous function on [0,1]. Then we define unification Bernstein type operator as follows:

$$\mathbb{S}_n(f(x)) = \sum_{j=0}^{n} f\left(\frac{j}{n}\right)\mathfrak{g}_n(j,x), \tag{2.2}$$

where $x \in [0,1]$, n is a positive integer.

Setting $f(x) = x$ in (2.2), then we have

$$\mathbb{S}_n(x) = \sum_{j=0}^{n}\frac{j}{n}\binom{n}{j}x^j(1-x)^{n-j}.$$

From the above equation, we get

$$\mathbb{S}_n(x) = x\sum_{j=0}^{n}\mathfrak{g}_{n-1}(j-1,x).$$

3. Fundamental relations of the polynomials $\mathfrak{S}_n(bs,x)$

By using generating function of $\mathfrak{S}_n(bs,x)$, in this section we give derivative of $\mathfrak{S}_n(bs,x)$ and recurrence relation of $\mathfrak{S}_n(bs,x)$.

Theorem 2. *Let $x \in [0,1]$. Let b, n and s be nonnegative integers with $n \geqslant bs$. Then*

$$\frac{d}{dx}\mathfrak{S}_n(bs,x) = n(\mathfrak{S}_{n-1}(bs-1,x) - \mathfrak{S}_{n-1}(bs,x)). \tag{3.1}$$

Proof. By differentiating the generating function in (2.1) with respect to x, we have