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Nonsmooth minimax programming under locally Lipschitz (Φ, ρ)-invexity

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ABSTRACT

Minimax programming problems involving locally Lipschitz (Φ, ρ)-invex functions are considered. The parametric and non-parametric necessary and sufficient optimality conditions for a class of nonsmooth minimax programming problems are obtained under nondifferentiable (Φ, ρ)-invexity assumption imposed on objective and constraint functions. When the sufficient conditions are utilized, parametric and non-parametric dual problems in the sense of Mond–Weir and Wolfe may be formulated and duality results are derived for the considered nonsmooth minimax programming problem. With the reference to the said functions we extend some results of optimality and duality for a larger class of nonsmooth minimax programming problems.

1. Introduction

Optimization problems, in which both a minimization and a maximization process are performed, are known in the area of mathematical programming as minimax (minmax) problems. Problems of this type arise frequently in the area of game theory, in economics, in best approximation theory, and in a great variety of situations involving optimal decision making under uncertainty. Minimax programming problems have been the subject of immense interest in the past few years. Necessary optimality conditions in the form of Lagrange multipliers for finite-dimensional constrained minimax problems have been originally investigated by Danskin [16] and Bram [11]. Making use of a certain generalized version of Gordan's theorem of the alternative, Schmitendorf [29] derived necessary optimality conditions for so-called static minimax problems. Several authors have been interested recently in the optimality conditions and duality results for different classes of minimax programming problems. Chew [13] studied minimax programming problems with pseudolinear functions. Tanimoto [30] obtained duality theorems for some minimax type problems under convexity assumptions. The importance of minimax models and methods is well known in a great variety of optimal decision making situations. Weir and Mond [26], and also Bector and Bhatia [6] established sufficient optimality conditions and duality results for minimax programming problems with the assumption of pseudo-convexity on the functions involved. Crouzeix et al. [15] have shown that the minimax fractional programming problem can be solved by solving a minimax nonlinear parametric program. Zalmai [33] extended the results established by Bector and Bhatia [6] and Schmitendorf [29] for a class of minimax programming problems in Banach spaces. On the other hand, Bhatia and Jain [10] derived optimality sufficient conditions for a general minimax programming problem under non-differentiable pseudo-convexity assumptions using pseudo-convexity in terms of classical Dini derivatives. Further, they introduced a dual in terms of Dini derivatives for a general minimax programming problem and established duality results. Mehra and Bhatia [23] proved optimality conditions and various duality results in the sense of Mond–Weir for a static minmax programming problem in terms of the right derivative of the functions involved with respect to the same arc. Bector et al. [8] obtained sufficient optimality conditions and duality results for a class of minimax programming problems under V-invexity assumptions on objective

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and constraint functions. Later, Antczak [2] established parametric and non-parametric sufficient optimality conditions and duality results for a class of minimax programming problems involving differentiable (p,r)-invex functions. In [24], Mishra et al. established sufficient optimality conditions and duality results for a minimax programming problem under (p,r)-Type I assumptions.

Recently, Antczak and Stasiak [4] generalized the definition of (Φ,ρ)-invexity notion introduced by Caristi et al. [12] for differentiable optimization problems to the case of mathematical programming problems with locally Lipschitz functions. They proved sufficient optimality conditions and duality results for nondifferentiable optimization problems involving locally Lipschitz (Φ,ρ)-invex functions.

In this paper, we consider a class of nonsmooth minimax programming problems in which functions involved are locally Lipschitz. Both parametric and nonparametric sufficient optimality conditions are established for this class of minimax problems involving locally Lipschitz (Φ,ρ) -invex functions. Subsequently, the necessary and sufficient optimality conditions are utilized as a basis for proving appropriate duality theorems for both parametric and parameter-free duality models constructed for the considered nonsmooth minimax programming problem. However, for obtaining sufficient optimality conditions and duality results, we make use of a locally Lipschitz (Φ,ρ)-invex concept. We illustrate the established results by a suitable example of a nonsmooth minimax programming problem involving locally Lipschitz (Φ,ρ)-invex functions. It turns out that to prove optimality and duality results in the considered nonsmooth minimax programming problem are not applicable sufficient optimality conditions established under generalized invexity notions existing in the literature. In this way, we extend the results of optimality and duality established earlier in the literature for a larger class of nonconvex nonsmooth minimax programming problems.

2. Nondifferentiable (Φ,ρ)-invexity

In this section, we provide some definitions and some results that we shall use in the sequel. Throughout this section, X is a nonempty subset of R^n.

Definition 1. A real-valued function $f : X \to R$ is said to be locally Lipschitz on X if, for any $x \in X$, there exist a neighborhood U of x and a positive constant $K_x > 0$ such that, for every $y, z \in U$,

$$|f(y) - f(z)| \leqq K_x\|y - z\|.$$

Definition 2 [14]. The Clarke generalized directional derivative of a locally Lipschitz function $f : X \to R$ at $x \in X$ in the direction $v \in R^n$, denoted $f^0(x; v)$, is given by

$$f^0(x;v) = \limsup_{\substack{y\to x\\ \lambda\downarrow 0}} \frac{f(y+\lambda v) - f(y)}{\lambda}.$$

Definition 3 [14]. The Clarke generalized subgradient of a locally Lipschitz function $f : X \to R$ at $x \in X$, denoted $\partial f(x)$, is defined as follows:

$$\partial f(x) = \{\xi \in R^n : f^0(x;v) \geqslant \xi\ v \text{ for all } v \in R^n\}.$$

It follows that, for any $v \in R^n$,

$$f^0(x;v) = \max\{\xi^T v : \xi \in \partial f(x)\}.$$

Definition 4. Let $f : X \to R$ be a locally Lipschitz function on X. A point $u \in X$ is said to be a stationary point of f if $0 \in \partial f(u)$.

The following result was given by Clarke [14].

Theorem 5. *If a locally Lipschitz function $f : X \to R$ attains a local minimum or maximum at $\bar{x}$, then $0 \in \partial f(\bar{x})$.*

Proposition 6 [14]. *Let $f_i : X \to R$, $i = 1, \ldots, k$, be locally Lipschitz functions on X. Then, for any scalars α_i, $i = 1, \ldots, k$, one has*

$$\partial\left(\sum_{i=1}^{k}\alpha_i f_i\right)(x) \subset \sum_{i=1}^{k}\alpha_i \partial f_i(x).$$

Now, we recall the definition of a locally Lipschitz (Φ,ρ)-invex function introduced by Antczak and Stasiak [4]. This class of nondifferentiable generalized invex functions was introduced as a generalization of differentiable (Φ,ρ)-invexity notion defined by Caristi et al. [12]. The main tool used in the definition of a locally Lipschitz (Φ,ρ)-invex function is a Clarke generalized subgradient (see Definition 3).