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B-splines methods with redefined basis functions for solving fourth order parabolic partial differential equations

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ABSTRACT

In this work, we discuss two methods for solving a fourth order parabolic partial differential equation. In Method-I, we decompose the given equation into a system of second order equations and solve them by using cubic B-spline method with redefined basis functions. In Method-II, the equation is solved directly by applying quintic B-spline method with redefined basis functions. Stability of these methods have been discussed. Both methods are unconditionally stable. These methods are tested on four examples. The computed results are compared wherever possible with those already available in literature. We have developed Method-I for fourth order non homogeneous parabolic partial differential equation from which we can obtain displacement and bending moment both simultaneously, while Method-II gives only displacement. The results show that the derived methods are easily implemented and approximate the exact solution very well.

1. Introduction

Consider the problem of undamped transverse vibrations of a flexible straight beam in such a way that its supports do not contribute to the strain energy of the system and is represented by the fourth order parabolic partial differential equation

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = f(x,t), \quad 0 \leqslant x \leqslant 1, \quad t > 0, \tag{1}$$

subject to initial conditions:

$$\left.\begin{aligned} u(x,0) &= g_0(x), \quad 0 \leqslant x \leqslant 1, \\ u_t(x,0) &= g_1(x), \quad 0 \leqslant x \leqslant 1, \end{aligned}\right\} \tag{2}$$

and boundary conditions are

$$\left.\begin{aligned} u(0,t) &= f_0(t), \quad t \geqslant 0, \\ u(1,t) &= f_1(t), \quad t \geqslant 0, \\ u_{xx}(0,t) &= p_0(t), \quad t \geqslant 0, \\ u_{xx}(1,t) &= p_1(t), \quad t \geqslant 0, \end{aligned}\right\} \tag{3}$$

where u is the transverse displacement of the beam, t and x are time and distance variables and $f(x,t)$ is dynamic driving force per unit mass.

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Numerical solution of (1) based on finite difference method after decomposition into a system of second order equation have been proposed successfully by Collatz [5], Conte [6], Crandall [7], Richtmyer and Morton [18], Evans [11], Todd [20] and Jain et al. [14]. Fairweather and Gourlay [12] derived an explicit and implicit scheme which were based on the semi-explicit method of Lees [15] and high accuracy method of Douglas [10] respectively. Danaee and Evans [8] proposed a stable computational scheme which was based on Hopscotch algorithm on decomposed form of the equation. Aziz et al. [3] developed a three level scheme based of parametric quintic spline and stability analysis had been carried out. Caglar and Caglar [4] have proposed a fifth degree B-spline method to solve (1)–(3) and have shown that their scheme produces accurate results. Recently, Rashidinia and Mohammadi [17] report a new three level implicit method based on sextic spline which is used to solve fourth order parabolic partial differential equation with variable coefficients.

In this paper, two numerical methods are developed for solving homogeneous and non-homogeneous fourth order parabolic partial differential equation. In Section 2, we present the process of decomposition of (1) into a system of two second order equations. This system is then solved by cubic B-spline collocation method with redefined basis functions [1,21] in Section 3. In Section 4, we describe the process of implementation of our scheme on reduced form of the equation. In Section 5, numerical stability of the scheme has been established by matrix method [8]. In Section 6, we present the way to obtain the initial state which is required to start our scheme I. In Section 7, we present Method-II, based on quintic B-spline collocation method with redefined basis function [1,16,22] for non- homogeneous equation. In Section 8, implementation process of method II is described. In Section 9, numerical stability of our scheme has been established by using von–Neumann method and Routh–Hurwitz criterion [9]. In Section 10, we present the way to obtain the initial state which is required to start our scheme II. In Section 11, we have taken 4 examples and implement our methods to obtain approximate numerical solution. Finally, in Section 12 conclusion of implementation and superiority of our methods based on numerical results of problems is mentioned.

2. Method I: Cubic B-splines collocation method with redefined basis functions

We solve Eq. (1) by introducing two new variables Φ and Ψ defined by

$$\Phi = \frac{\partial u}{\partial t}, \quad \Psi = \frac{\partial^2 u}{\partial x^2}. \tag{4}$$

Now, (1) can be rewritten as two simultaneous partial differential equation of the form

$$\left.\begin{aligned} \frac{\partial \Phi}{\partial t} &= -\frac{\partial^2 \Psi}{\partial x^2} + \mathrm{f}, \\ \frac{\partial \Psi}{\partial t} &= \frac{\partial^2 \Phi}{\partial x^2}. \end{aligned}\right\} \tag{5}$$

Now, (5) can be written as second order system

$$\frac{\partial \Omega}{\partial t} = A\frac{\partial^2 \Omega}{\partial x^2} + F, \tag{6}$$

where $\Omega = \begin{bmatrix} \Phi \\ \Psi \end{bmatrix}$, $\mathrm{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\mathrm{F} = \begin{bmatrix} \mathrm{f} \\ 0 \end{bmatrix}$

Since and $A + A' = 0$ and $A^{-1} = -A$, (6) is a Schrödinger type of partial differential equation.

Initial conditions (2) and boundary conditions (3) now become

$$\begin{aligned} &\Omega(x,0) = w_0(x), \\ &\Omega(0,t) = a_0(t), \Omega(1,t) = b_0(t). \end{aligned} \tag{7}$$

3. Description of Method-I

In collocation method the approximate solution can be written as a linear combination of basis functions which constitute a basis for the approximation space under consideration.

We consider a mesh $0 = x_0 < x_1, \ldots, x_{N-1} < x_N = 1$ as a uniform partition of the solution domain $0 \leqslant x \leqslant 1$ by the knots x_i with $h = x_i - x_{i-1}$, $i = 1, \ldots, N$.

Our numerical treatment for solving Eq. (6) using the collocation method with cubic B-spline is to find an approximate solution $\Omega(x,t)$ to the exact solution $\Omega(x,t)$ in the form:

$$\Omega(x,t) = \sum_{i=-1}^{N+1} c_i(t) B_i(x) \tag{8}$$

where $c_i(t)$ are time dependent quantities to be determined from the boundary conditions and collocation from the differential equation.