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Adaptive control of linear time invariant systems via a wavelet network and applications to control Lorenz chaos

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ABSTRACT

Compactly supported orthogonal wavelets have certain properties that are useful for controller design. In this paper, we explore the mechanism of a wavelet controller by integrating the controller with linear time-invariant systems (LTI). A necessary condition for effective control is that the compact support of the wavelet network covers the state space where the state trajectories stay. Closed-form bounds on the design parameters of the wavelet controller are derived, which guarantee asymptotic stability of wavelet-controlled LTI systems. The same wavelet controller is then applied to the Lorenz equations. The control objective is to stabilize the Lorenz system well into its normally chaotic region at one of its equilibria.

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1. Introduction

The applications of wavelet theory have been thriving in many branches of the engineering sciences during the past decade. Wavelet-based multiresolution analysis is superior to the Fourier analysis because of its "zoom-in" (localization) property in both time (space) and frequency domains, which is powerful in capturing nonlinear behavior in dynamical systems. As such, wavelet theory has drawn a great deal of attention from the control community on the topics of system identification and adaptive controller designs.

There are a number of papers dedicated to the application of wavelet analysis in the study of nonlinear dynamical systems, mostly in system identification, signal reconstruction, and solution of nonlinear differential/integral equations. Xin and Sano investigated the system identification of an FIR model via wavelet packets filter bank using an adaptive scheme [1]. The wavelet filter bank approach is shown to be effective in dealing with ill-conditioned process for reconstructing the filter coefficients. Sureshbabu and Farrel [2] developed a system identification algorithm using orthogonal wavelets. Cao et al. [3] applied wavelets to predict chaotic time series from nonlinear dynamical systems. Haar wavelet with variable stepsize is proposed in [4] to solve integral and differential equations. The Haar wavelet based approach is effective in adapting the solution process to irregularities in the solution. Hashish et al. [5] suggested a wavelet-Galerkin method to solve nonhomogeneous 2-D heat equation in finite rectangular domains. The test functions for the two-dimensional Galerkin method are scaling functions associated with certain wavelets. The method is adaptive to various boundary conditions with simplified geometry constraints on the domain of the testing function. After Zhang and Benveniste [6] introduced a new prediction technique, so-called wavelet networks, a number of adaptive control techniques based on the idea of wavelet networks were developed [7–11]. A common feature of these wavelet-based adaptive control methods is that the controller consists of a negative state feedback (proportional controller) and a nonlinear component approximated by a wavelet network. The nonlinear component of the controller is used to eliminate the nonlinearity of the original dynamical system

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so that the nonlinear system is reduced to either a linear system or a much simplified nonlinear system. Hence, the classical controller design approaches can be applied [10]. There are other variants of wavelet-based adaptive controller designs in the literature. In [12], a wavelet network modeled only with scale functions maps the input space of the uncertain nonlinear model to output space in wavelons, followed by an implementation of nonlinear adaptive predictive control. It is shown in [13] that a wavelet controller is effective in eliminating repetitive errors in disk drives by accurately approximating periodic functions so that the disks function smoothly without the presence of disturbances. A wavelet adaptive backstepping control method is proposed in [14], in which the wavelet network serves as the system identifier in the backstepping controller by taking advantage of the adaptive learning capability of the wavelet network. The output of the wavelet network is used in constructing the robust controller recursively so that the system achieves the tracking performance at the desired attenuation level. The proposed wavelet-based controller design in this paper is different from the existing adaptive controller designs. Based on our observations, many controllers, such as a regulator, resemble the waveform of a wavelet. More precisely, many controllers as functions belong to $L^2(\mathbb{R}^n)$, i.e. the space of square-integrable functions. The wavelet networks are widely used in system identification and control in which the wavelons are dilated and translated variants of the mother wavelet, which are localized in both space and frequency domains. This architecture has a powerful approximation property in approximating any $L^2(\mathbb{R}^n)$ functions to arbitrary precision by linear combinations of wavelets [15]. This motivates us to represent a controller in terms of a subset of wavelet basis functions. Wang et al. [16] adopted this idea to control chaos in nonlinear systems. However, they merely reported numerical simulation results without providing qualitative analysis on the stability of the wavelet-controlled nonlinear systems. It is the purpose of this paper to present a qualitative analysis on the mechanism and stability properties of a wavelet approximated regulator when it is applied to LTI systems.

The rationale for our choice of using LTI systems to study the characteristics of a wavelet controller is two fold: Firstly, a LTI system is simple in structure, yet a benchmark system in the area of control. This allows us to study the mechanism of a wavelet controller without being sidetracked by the complexity of the dynamical system itself; Secondly, whenever the control objective is to stabilize a nonlinear system in the vicinity of one of its equilibria, such as a regulator, a common approach is to linearize the original system at the specific equilibrium point, which leads to a LTI model.

The rest of this paper is organized as follows. In Section 2, the structure of a wavelet controller is discussed. Stability bounds on the design parameters of a wavelet controller are presented in the main theorem of this paper, followed by a proof. Numerical simulations for wavelet-controlled LTI systems are reported in Section 3. Applications of a wavelet controller to control Lorenz chaos are discussed in Section 4, and the main results are summarized in the conclusion section.

2. Wavelet-based control for LTI systems

A wavelet function [15], $\psi \in L^2(\mathbb{R}^n)$, satisfies

$$\int_{R^n} \frac{|\Psi(\omega)|^2}{|(\omega)|} d\omega < \infty, \tag{1}$$

where $\Psi(\omega)$ is the Fourier transform of $\psi(x)$. Both $\psi(x)$ and $\Psi(\omega)$ are compactly supported or nearly compactly supported in their respective domains. With appropriate shifts (t_k) and dilations (d_l) applied to the (mother) wavelet function $\psi(x)$, one obtains a denumerable family of wavelets

$$\Phi = \left\{ \begin{array}{ll} \psi_{lk}(x)|\psi_{lk}(x) = \det(D_l)^{1/2}\psi(D_l x - t_k), & t_k \in \mathrm{R}^n, \\ D_l = diag(d_l), d_l \in \mathrm{R}^n_+, & l,k \in \mathrm{Z} \end{array} \right\} \tag{2}$$

satisfying the frame property: there exist two constants, $A > 0$ and $0 < B < \infty$, such that, for all $f \in L^2(\mathrm{R}^n)$, the following inequalities hold:

$$A\|f\|^2 \leqslant \sum_{\phi\in\Phi} |<\phi,f>|^2 \leqslant B\|f\|^2. \tag{3}$$

As a result, the wavelet family Φ is dense in $L^2(\mathbb{R}^n)$.

Under the assumption that a wavelet controller u is in $L^2(R^n)$, we will represent a controller in terms of a subset of wavelets in (2). It is noted that a wavelet controller is calculated from the sampled values of state variables. We will focus on the stability issues of the real-time implementation of a wavelet controller. We first identify the design parameters as the sampling interval T and the adaptive gain η from the iterative Eq. (7). We will prove that the wavelet-controlled LTI system is asymptotically stable at the origin. As a result of the proof, we obtain bounds on the design parameters of a wavelet controller. The significance of the proof is that we provide sufficient conditions and computable bounds for the design parameters that guarantee stability of the wavelet-controlled LTI plant. Those bounds can be used in real-time to update the parameters online so that the dynamical system is stabilized.

The state space representation of a LTI plant with a wavelet controller is given by

$$\dot{x} = Ax + Bu, \tag{4}$$