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A joint optimal ordering and delivery policy for an integrated supplier–retailer inventory model with trade credit and defective items

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ABSTRACT

In this paper, we propose an integrated supplier–retailer inventory model in which both supplier and retailer have adopted trade credit policies, and the retailer receives an arriving lot containing some defective items. The customer's market demand rate depends on the length of the credit period offered by retailer. Our objective is to determine the retailer's optimal order cycle length, the order quantity, and the optimal number of shipments per production run from the supplier to the retailer so that the entire supply system has maximum profit. We develop an algorithm to find the optimal solution for the supply chain. Several numerical examples are provided to illustrate the theoretical results, and sensitivity analysis of major parameters including the defective rate in a production batch, the retailer's trade credit period and the customer's trade credit period in the model are presented.

1. Introduction

In most of the early literature dealing with inventory problems, emphases were placed on either the retailer's or the supplier's perspective to minimize the cost and/or to maximize the profit. Recently, the integrated inventory models have become more and more important, because the supplier and the retailer can increase their mutual benefit through strategic cooperation. Goyal [1] developed an integrated model for a single supplier-single retailer system to find the optimal order quantity of the retailer so that the total cost at the system is minimized. Monahan [2] examined the quantity discount problem from the supplier point of view and obtained the minimum cost of the entire supply chain. Banerjee [3] presented a joint economic-lot-size model where a supplier produces for a retailer to order on a lot-for-lot basis. Goyal [4] generalized Banerjee's [3] model by relaxing the assumption of the lot-for-lot policy of the supplier and illustrated that the inventory cost can be reduced significantly if the supplier's economic production quantity is a positive integer multiple of the retailer's purchase quantity. Lu [5] assumed that the supplier's production rate is greater than the demand rate, and the delivery starts as soon as the quantity ordered by the retailer is produced, and later on goods are delivered on a lot-for-lot basis. Goyal [6] relaxed the lot-for-lot policy and assumed that if the demand is constant, shipment sizes will increase according to the ratio of production rate and demand rate. Goyal and Nebebe [7] proposed the first shipment to be smaller and is followed by shipments of equal size. Recently, Ouyang et al. [8] proposed an integrated inventory model with quality improvement and lead

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time reduction. Other related studies of the integrated inventory model include Yang and Wee [9], Yang et al. [10], Wee and Chung [11], Teng et al. [12], and so on.

In the traditional inventory models, the theme of defective items is always ignored. However, defective items can by caused by the incomplete production process and/or damage in transit. And the number of defective items will influence the on-hand level and the number of orders in the inventory system. In addition, if the retailer sells defective items without inspection, the customers will complain, return the goods, or even never come back. In all cases, substantial costs are incurred. Already there are some scholars who have studied and developed various analytical inventory models about defective items. Porteus [13] and Rosenblatt and Lee [14] are among the first ones who analyzed a significant relationship between quality imperfection and lot size. Next, Paknejad et al. [15] proposed a modified EOQ model with stochastic demand, and the model included the number of defective items in a lot as a random variable. In each delivery, the defective items will be found in each lot and sent back to the supplier in the delivery time of the next batch. Salameh and Jaber [16] presented an EPQ model with defective items, and they assumed that the production rate for the non-defective items is greater than the demand rate. Ouyang and Chang [17] presented an investment in quality improvement inventory model involving defective items production process with controllable lead time. There are more papers related to this issue of defective items such as Chung and Hou [18], Hou [19], Rahim and Al-Hajailan [20], Lin [21], Wee et al. [22], Sarkar [23], and Barzoki et al. [24], etc.

Furthermore, in practical situations, in order to motivate retailer to increase order quantity and market share, the supplier often offers a trade credit to the retailer, that is, the retailer may receive goods or services without having to pay until sometime later. Haley and Higgins [25] first presented an inventory model with the permissible delay in payments. Ferris [26] derived a transactions theory of trade credit use from the motives of trading partners to economize on the joint costs of exchange. Kingsman [27] considered the effects of different ways of payment on ordering and stocking. Goyal [28] established an EOQ inventory model with interest earned and paid under the condition of permissible delay in payments. Aggarwal and Jaggi [29] extended Goyal's [28] model to include deteriorating items. Jamal et al. [30] further generalized this issue with allowable shortages. Buzacott and Zhang [31] proposed an inventory management to incorporate asset-based financing into production decisions. In their paper, the retailers buy a product from the suppliers and then sell it to the customers in which the retailers require asset-based financing by bank to purchase product from the suppliers. Among other relative inventory-financing issues studies were Hill and Riener [32], Abad and Jaggi [33], Chen and Kang [34], Huang and Hsu [35], Ho et al. [36] and Thangam and Uthayakumar [37].

Because of changing of the business environment, the delay payments of trade credit change with each passing day. There exist numerous interesting and relevant papers related to trade credits, but most assume that the supplier offers a trade credit to the retailer. However, the retailer wishes to motivate the customer's demand rate and to reduce the on-hand stock cost, and offers a trade credit to the customers. Huang [38] considered an EOQ inventory model in which both supplier and retailer have adopted trade credit policies. Su et al. [39] developed an integrated supplier-retailer inventory model in which the customer's demand for goods is positively correlated to the credit period offered by the retailer. They discussed how to obtain optimal order quantity, shipping, and inventory policy.

In this paper, we considered the integrated supplier-retailer inventory model in which both supplier and retailer have adopted a trade credit strategy, involving defective items, and the demand rate varies according to the length of the customer's trade credit period. This study aims to find the optimal retailer's replenishment cycle length, the order quantity, and the optimal number of shipments per production run from the supplier to the retailer, so that the entire supply chain has maximum profit. An algorithm is developed to determine the optimal solution. Finally, numerical examples are presented to illustrate the solution procedure, and sensitivity analysis of major parameters involved in the model is also made.

2. Notation and assumptions

In this paper, the mathematical model is developed on the basis of the following notation and assumptions.
Notation:

P	The supplier's production rate.
D	The retailer's demand rate.
K	The supplier's setup cost per order.
A	The retailer's ordering cost per unit ordered.
F	Transportation cost per delivery.
h_v	The supplier's holding cost per item per unit time.
h_{b_1}	The retailer's holding cost per non-defective item per unit time, excluding interest charges.
h_{b_2}	The retailer's holding cost per defective item per unit time (including treatment cost), excluding interest charges, $h_{b_2} \leqslant h_{b_1}$.
s	The retailer's unit screening cost.
x	The retailer's screening rate per order.
γ	The supplier's defective rate in a production batch, is given, $0 \leqslant \gamma < 1$.