



Comparison between Darcy and Brinkman laws in a fracture

Eduard Marušić-Paloka^a, Igor Pažanin^{a,*}, Sanja Marušić^b

^a Department of Mathematics, Faculty of Science, University of Zagreb, Bijenička 30, Zagreb, Croatia

^b Faculty of Transport and Traffic Engineering, University of Zagreb, Vukelićeva 4, Zagreb, Croatia

ARTICLE INFO

Keywords:

Brinkman law

Darcy law

Porous medium

Asymptotic expansion

ABSTRACT

Different laws are used for modeling flows in porous media. In this paper, we focus on Brinkman and Darcy law. We derive them from microscopic equations by upscaling, compare them and estimate the error made by their application. Our results justify the use of Brinkman law.

© 2012 Elsevier Inc. All rights reserved.

1. Introduction

Frequently in applications the equations given by first principles are too complicated to be solved in engineering practice. In such situations empiric laws are a good substitute for original physical laws. Nevertheless, it is of practical interest to theoretically compare empiric laws with those obtained from the first principles in order to justify the application of the empiric laws, see what are their limitations and, if possible, to estimate the error made by their application. Typical example of such situation is the fluid flow through porous medium. Conservation of mass and momentum lead to the Navier–Stokes system in a complex geometry of porous medium. It accurately describes the flow locally, but it is useless for macroscopic simulations of the flow due to the complexity of the geometry.

Different empiric laws are used to describe the filtration of a fluid through porous medium. We focus on the **Darcy law** [1]:

$$\mathbf{v} = \frac{1}{\mu} \mathbf{K}(\mathbf{F} - \nabla p) \Rightarrow \mu \mathbf{B} \mathbf{v} + \nabla p = \mathbf{F}, \quad \mathbf{B} = \mathbf{K}^{-1},$$

$$\operatorname{div} \mathbf{v} = 0$$

and the **Brinkman law** [2]:

$$\mu \mathbf{B} \mathbf{v} - \nu \Delta \mathbf{v} + \nabla p = \mathbf{F},$$

$$\operatorname{div} \mathbf{v} = 0.$$

Darcy law is the simplest and, by far, the most popular one, due to its simplicity. It states that the filtration velocity of the fluid is proportional to the difference between the body force and the pressure gradient. The tensor $\mathbf{K}$ appearing in the relation is called the permeability of the medium. In the sequel, we prefer to express the law in terms of the resistance tensor $\mathbf{B} = \mathbf{K}^{-1}$. The simplicity of Darcy law can sometimes be its main drawback. Indeed, being a first order PDE for the velocity, it cannot sustain the no-slip condition on an impermeable wall or a transmission condition on the contact with free flow. That motivated H. Brinkmann in 1947 to modify the Darcy law in order to be able to impose the no-slip boundary condition on an obstacle submerged in porous medium. He assumed large permeability to compare his law with experimental data and assumed that the second viscosity ν equals the physical viscosity of the fluid μ . If those two viscosities were equal, the

* Corresponding author.

E-mail addresses: emarusic@math.hr (E. Marušić-Paloka), pazanin@math.hr (I. Pažanin), marusics@fpz.hr (S. Marušić).

Brinkman law could formally be obtained from the Stokes system describing the microscopic flow, by adding a linear term $\mathbf{B}\mathbf{v}$, representing the resistance to the flow. That would be a reasonable physical interpretation of the Brinkman law, if $\mu = \nu$. As we shall see later, the two viscosities are, in general, not equal.

Many papers have been devoted to the subject since then. We mention here only some of them that inspired our work. In 1970, Beavers et al. [3] compared ν and μ for different artificial porous materials using experimental data. In 1974, Neale and Nader [4], formally derived Beavers and Joseph condition for contact of a porous medium and a free flow, from Brinkman law. In 1982–1983, Sanchez-Palencia [5] and Levy [6] derived Brinkman law from Stokes system using formal homogenization and asymptotic analysis for flow through an array of small particles. In 1991, Allaire [7] rigorously proved their result using ideas proposed by Cioranescu and Murat [8] for obstacles of size ε^3 in ε -periodic medium. Recently Auriault et al. [9] formally computed correctors for the Darcy law and derive the Brinkman's correction as a lower order term in an asymptotic expansion attributed to the flow.

Brinkman model is used in numerical applications as it allows to resolve problems with boundary conditions on impermeable boundary (see e.g. [10,11]) as well as on the interface between porous medium and an open fluid domain (see e.g. [12]). Since the Brinkman term appears to be small, frequently for such numerical simulations an artificial term of the Brinkman form is simply added to the Darcy equations. In view of that, this work justifies the use of the Brinkman corrector to the Darcy system. Moreover, we provide its explicit form of in the case of the flow through a thin fracture together with the sharp error estimate and that represents our main contribution.

2. Flow through a fracture

In the present paper we model the simplified situation in which the fluid flows through a fracture. Fracture can be seen as an oversimplified version of porous medium. The obvious advantage of such framework is that the permeability can be explicitly computed, which is important for what we want to point out. Most of our computations can be repeated in a periodic porous medium as well.

2.1. The geometry

First we formally describe the simple geometry of a thin fracture. Let $\mathcal{O} \subset \mathbf{R}^2$ be a bounded domain and $h : \overline{\mathcal{O}} \rightarrow]0, +\infty[$ a smooth positive function. Let $\varepsilon > 0$ be a small parameter. We study the fluid flow through a thin domain of the form

$$\Omega^\varepsilon = \{x = (x_1, x_2, x_3) \in \mathbf{R}^3 : x' = (x_1, x_2) \in \mathcal{O}, 0 < x_3 < \varepsilon h(x')\}.$$

For the sake of simplicity, lower surface of the fracture is assumed to be plane, while the upper is rough with roughness described by the function h . The average fracture thickness is ε , while the pointwise thickness is given by $H^\varepsilon = \varepsilon h$.

2.2. The equations

We assume that the flow is slow enough so that the inertial effects can be neglected and choose the Stokes system for the governing equations:

$$-\mu \Delta \mathbf{u}^\varepsilon + \nabla p^\varepsilon = \mathbf{F} \quad \text{in } \Omega^\varepsilon, \quad (1)$$

$$\operatorname{div} \mathbf{u}^\varepsilon = 0 \quad \text{in } \Omega^\varepsilon, \quad (2)$$

$$\mathbf{u}^\varepsilon = 0 \quad \text{for } x_3 = 0, H^\varepsilon. \quad (3)$$

The vector field $\mathbf{u}^\varepsilon$ denotes the fluid velocity whereas the pressure is given by the scalar field p^ε . The superscript ε is added in order to stress the dependence of the solution on the small parameter. The viscosity μ is a positive constant, while $\mathbf{F} = \mathbf{F}(x')$ stands for a given forcing term such that $f_3 = \mathbf{F} \cdot \mathbf{k} = 0$. Here and in the sequel we denote by $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ the standard Cartesian basis. For the moment we do not discuss the boundary conditions on the lateral boundary

$$\Gamma^\varepsilon = \{x \in \mathbf{R}^3 : x' \in \partial\mathcal{O}, 0 < x_3 < H^\varepsilon(x')\}.$$

Our goal is to find an effective law describing the behavior of the flow in a thin fracture Ω^ε .

3. Formal derivation

Following the multiscale expansion technique, the velocity $\mathbf{u}^\varepsilon$ and the pressure p^ε are looked for in the form of two-scale asymptotic expansions in powers of ε :

$$\mathbf{u}^\varepsilon = \varepsilon^2 \mathbf{u}^0(x', y) + \varepsilon^3 \mathbf{u}^1(x', y) + \varepsilon^4 \mathbf{u}^2(x', y) + \dots, \quad (4)$$

$$p^\varepsilon = p^0(x') + \varepsilon p^1(x', y) + \varepsilon^2 p^2(x', y) + \dots \quad (5)$$

We denote the fast variable by $y = \frac{x_3}{\varepsilon}$. The leading powers in expansions for $\mathbf{u}^\varepsilon$ and p^ε are suggested by the *a priori* estimates which can be easily derived from the corresponding weak formulation. Furthermore, notice that $p^0 = p^0(x')$ since

Download English Version:

<https://daneshyari.com/en/article/4630863>

Download Persian Version:

<https://daneshyari.com/article/4630863>

[Daneshyari.com](https://daneshyari.com)