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Effect of the rotation on a non-homogeneous infinite cylinder of orthotropic material

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ABSTRACT

The present investigation is concerned with a study effect of non-homogeneous on the elastic stresses in rotating orthotropic infinite circular cylinder subjected to certain boundary conditions. Closed form stress solutions are obtained for rotating orthotropic cylinder with constant thickness for three cases: (1) a solid cylinder; (2) cylinder mounted on a circular rigid shaft; and (3) cylinder with a circular hole at the center. Analytical expressions for the components of the displacement and the stress in different cases are obtained. The effect of the rotation and non-homogeneity on the displacement and stress are studied. Numerical results are given and illustrated graphically for each case is considered. The effects rotating and non-homogeneity are discussed. Comparisons are made with the results predicted in the presence and absence of rotation.

1. Introduction

In the past, accident of rotating cylinder wheels due to flexural vibration has frequently occurred in rotodynamic machinery such as steam turbines and gas turbines. At the present time applied mathematicians are exhibiting considerable interest in dynamical methods of elasticity, since the usual quasi-static approach ignores certain very important features of the problems under consideration. That approach is based on the assumption that the inertia terms may be omitted from the equations of motion. This assumption holds good only when the variations in stresses and displacements but there arise number of problems in engineering and technology. When this assumption may not hold good and the inertia terms in the equations of motion may have lead to cases of considerable mathematical complications. With increasing application of composite and orthotropic materials in rotating components of machinery and structures (e.g. flywheels, turbines, etc.). Abd-Alla et al. [1,2] investigated effect of the non-homogeneity on the composite infinite cylinder of isotropic and orthotropic material. El-Naggar et al. [3,4] studied the rotation of a non-homogeneous composite infinite cylinder of orthotropic material and the stresses in a rotation of a non-homogeneous composite cylinder of isotropic material. Ding, et al. [5,6] discussed analytical solution for the electroelastic dynamics of a nonhomogeneous spherically isotropic and a solution of a non-homogeneous orthotropic cylindrical shell for axisymmetric plane strain dynamic thermoelastic Hosseini, et al. [7] investigated thermoelastic analysis of a functionally graded rotating disk. Grigorenko and Rozhok [8] investigated equilibrium of elastic hollow inhomogeneous cylinders of corrugated elliptic cross-section. Eslami et al. [9] discussed thermal and mechanical stresses in a functionally graded thick sphere. Theotokoglou and Stampouloglou [10] investigated the radially nonhomogeneous elastic axisymmetric problem. Mahmoud [11] studied the wave propagation in cylindrical poroelastic dry bones. Abd-Alla and Mahmoud [12] solved magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylindrical under the hyperbolic

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heat conduction model, and Mahmoud [13] studied the effect of the non-homogeneity on wave propagation on orthotropic elastic media. Influences of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space is investigated by Abd-Alla et al. [14,15]. Mahmoud et al. [16] discussed effect of the rotation on the radial vibrations in a non-homogeneous orthotropic hollow cylinder. Grigorenko and Vlaikov [17] studied some problems of elasticity theory for orthotropic bodies of cylindrical form.

In the present paper, we are assuming that elastic constants and the density are power functions of the radial coordinate. The solutions have been obtained in analytical form in both cases, the stresses have been calculated. Finally, comparisons between both cases are clarified by figures. The stresses in rotating cylinder made of orthotropic materials will be found for the following three cases: (1) a solid cylinder; (2) a cylinder mounted on a circular rigid shaft; and (3) a cylinder with a circular hole at the center. The numerical results are discussed in detail for material of non-homogeneous and the rotation effects for different cases is considered, the results indicate that the effect of inhomogeneity and rotation are very pronounced.

2. Formulation of the problem

Let us consider the cylindrical coordinates (r,θ,z) with the z-axis coinciding with the axis of the cylinder. We consider the strains symmetric about the z-axis. We have only the radial displacement $u_r = u$, u is a function of r and t only, the circumferential displacement $u_\theta = 0$ and the longitudinal displacement $u_z = 0$, which are independent of θ and z. The stress–strain relations for cylindrically orthotropic material are given by Lekhnitskii [18] in the form. The analysis is based on the following assumptions: (1) the infinitesimal elasticity theory of an orthotropic body; (2) the materials are macroscopically homogeneous and cylindrically orthotropic; (3) stress–strain relations obey a generalized Hooke's law; (4) the condition of plane strain is used. In plane strain in the plane perpendicular to the z- axis, u is a function of r alone. The stresses components are given by:

$$\begin{aligned}
\sigma_{rr} &= c_{11}\frac{du}{dr} + c_{12}\frac{u}{r},\\
\sigma_{\theta\theta} &= c_{12}\frac{du}{dr} + c_{22}\frac{u}{r},\\
\sigma_{zz} &= c_{13}\frac{du}{dr} + c_{23}\frac{u}{r},\\
\tau_{r\theta} &= \tau_{rz} = \tau_{\theta z} = 0.
\end{aligned} \tag{1}$$

The equilibrium equation in the direction r is given by:

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) + \rho\Omega^2 r = 0, \tag{2}$$

where c_{ij} are the elastic constants, Ω is the uniform angular velocity and ρ is the density of the material of the cylinder. We characterize the elastic constants c_{ij} and the density ρ of non-homogeneous material by:

$$c_{ij} = \alpha_{ij} r^{2m} \quad at\ i = 1,2; \quad j = 1,2,3 \quad \text{and } \rho = \rho_0 r^{2m}, \tag{3}$$

where α_{ij}, ρ are constants and m is a rational number, the elastic matrix of the elastic constants is

$$\begin{pmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\
c_{12} & c_{22} & c_{23} & 0 & 0 & 0\\
c_{13} & c_{23} & c_{33} & 0 & 0 & 0\\
0 & 0 & 0 & c_{44} & 0 & 0\\
0 & 0 & 0 & 0 & c_{55} & 0\\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{pmatrix}.$$

3. Solution of the problem

We seek the solution of the equation (2) as:

$$\frac{d}{dr}(r\sigma_{rr}) - \sigma_{\theta\theta} + \rho\Omega^2 r^2 = 0, \quad \sigma_{rr} = f(r). \tag{4}$$

Substituting from Eqs. (1), (3) into Eq. (5) we obtain:

$$r^2\frac{d^2u}{dr^2} + (2m+1)r\frac{du}{dr} + \frac{(2m\alpha_{12} - \alpha_{22})}{\alpha_{11}}u = -\frac{\rho_0\Omega^2}{\alpha_{11}}r^3. \tag{5}$$

The complete solution of Eq. (6) is: