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Graph convergence for the $H(\cdot,\cdot)$-accretive operator in Banach spaces with an application ☆

Xi Li, Nan-jing Huang *

Department of Mathematics, Sichuan University, Chengdu, Sichuan 610064, PR China

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ABSTRACT

In this paper, a concept of graph convergence concerned with the $H(\cdot,\cdot)$-accretive operator is introduced in Banach spaces and some equivalence theorems between of graph-convergence and resolvent operator convergence for the $H(\cdot,\cdot)$-accretive operator sequence are proved. As an application, a perturbed algorithm for solving a class of variational inclusions involving the $H(\cdot,\cdot)$-accretive operator is constructed. Under some suitable conditions, the existence of the solution for the variational inclusions and the convergence of iterative sequence generated by the perturbed algorithm are also given.

1. Introduction

It is well known that the variational inclusion is an important and useful generalization of the variational inequality. Because of the wide applications to optimization and control, economic and transportation equilibrium, and engineering sciences, the variational inequalities and variational inclusions with applications have been intensively studied by many authors. One of the most important and interesting problems in the theory of variational inequality is the development of an efficient and implementable algorithm for solving the variational inequality. For the past years, many numerical methods have been developed for solving various classes of variational inequalities, such as the projection method and its variant forms, linear approximation, descent, and Newton's methods.

Recently, many authors have studied the perturbed algorithms for variational inequalities involving maximal monotone mappings in Hilbert spaces. Using the concept of graph-convergence for maximal monotone mappings and the equivalence between of graph-convergence and resolvent operator convergence considered by Attouch [1], they constructed some perturbed algorithm for variational inequality and proved the convergence of sequences generated by perturbed algorithms under some suitable conditions (see, for example [2–4,13,14,16,20]).

On the other hand, in 2001, Huang and Fang [17] were the first to introduce the generalized m-accretive mapping and give the definition of the resolvent operator for the generalized m-accretive mapping in Banach spaces. They also showed some properties of the resolvent operator for generalized m-accretive mappings in Banach spaces. Recently, Fang and Huang, Lan, Cho and Verma investigated several generalized operators such as H-accretive, (H,η)-accretive (A,η)-monotone mappings and (A,η)-accretive mappings (see, for example [8–11,16–18,21–25] and the references therein). Very recently, Zou and Huang [31] introduced a new concept of $H(\cdot,\cdot)$-accretive operators, which generalized the existing maximal monotone or accretive operators. They studied some properties of $H(\cdot,\cdot)$-accretive operators and defined resolvent operators associated

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* Corresponding author.
E-mail address: nanjinghuang@hotmail.com (N.-j. Huang).

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with $H(\cdot,\cdot)$-accretive operators. They also studied a class of variational inclusion using the resolvent operators associated with $H(\cdot,\cdot)$-accretive operators. Some related works, we refer to [5–7,12,15,26–28,30,32].

Motivated and inspired by the work mentioned above, in this paper, we introduce a concept of graph convergence for $H(\cdot,\cdot)$-accretive operators. We show some equivalence theorems between of graph-convergence and resolvent operator convergence the $H(\cdot,\cdot)$-accretive operator sequence in Banach space. As an application, we studied the perturbed algorithm for a class of variational inclusions involving $H(\cdot,\cdot)$-accretive operator in Banach spaces. Under some suitable conditions, we prove the existence of the solution for the variational inclusions and the convergence of the iterative sequence generated by the perturbed algorithm.

2. Preliminaries

Let X be a real Banach space with dual space X^*, $\langle\cdot,\cdot\rangle$ be the dual pair between X and X^*, and 2^X denote the family of all the nonempty subsets of X. The generalized duality mapping $J_q : X \to 2^{X^*}$ is defined by

$$J_q(x) = \{f^* \in X^* : \langle x, f^* \rangle = \|x\|^q, \|f^*\| = \|x\|^{q-1}\}, \quad \forall x \in X,$$

where $q > 1$ is a constant. In particular, J_2 is the usual normalized duality mapping. It is known that, in general, $J_q(x) = \|x\|^{q-1} J_2(x)$ for all $x \neq 0$ and J_q is single-valued if X^* is strictly convex. In the sequel, we always assume that X is a real Banach space such that J_q is single-valued. If X is a Hilbert space, then J_2 becomes the identity mapping on X.

The modulus of smoothness of X is the function ρ_X: $[0,\infty) \to [0,\infty)$ defined by

$$\rho_X(t) = \sup\left\{\frac{1}{2}(\|x+y\| + \|x-y\|) - 1 : \|x\| \leqslant 1, \|y\| \leqslant t\right\}.$$

A Banach space X is called uniformly smooth if

$$\lim_{t\to 0} \frac{\rho_X(t)}{t} = 0.$$

X is called q-uniformly smooth if there exists a constant $c > 0$ such that

$$\rho_X(t) \leqslant ct^q, \quad q > 1.$$

Note that J_q is single-valued if X is uniformly smooth. Concerned with the characteristic inequalities in q-uniformly smooth Banach spaces, Xu [29] proved the following result.

Lemma 2.1. *Let X be a real uniformly smooth Banach space. Then X is q-uniformly smooth if and only if there exists a constant $c_q > 0$ such that, for all $x, y \in X$,*

$$\|x+y\|^q \leqslant \|x\|^q + q\langle y, J_q(x)\rangle + c_q\|y\|^q.$$

From Lemma 2 of Liu [19], it is easy to have the following lemma.

Lemma 2.2. *Let $\{a_n\}$ and $\{b_n\}$ be two nonnegative real sequences satisfying*

$$a_{n+1} \leqslant ka_n + b_n$$

with $0 < k < 1$ and $b_n \to 0$. Then $\lim_{n\to\infty} a_n = 0$.

Definition 2.1. Let A, B: $X \to X$ and H: $X \times X \to X$ be three single-valued mappings.

(i) A is said to be accretive if

$$\langle Ax - Ay, J_q(x-y)\rangle \geqslant 0, \quad \forall x, y \in X;$$

(ii) A is said to be strictly accretive if A is accretive and

$$\langle Ax - Ay, J_q(x-y)\rangle = 0,$$

if and only if $x = y$;

(iii) A is said to be δ-strongly accretive if

$$\langle Ax - Ay, J_q(x-y)\rangle \geqslant \delta\|x-y\|^q.$$

(iv) A is said to be γ-Lipschitz continuous if there exists a constant $\gamma > 0$ such that

$$\|Ax - Ay\| \leqslant \gamma\|x-y\|, \quad \forall x, y \in X;$$