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Nonmonotone filter DQMM method for the system of nonlinear equations

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ABSTRACT

In this paper, we propose a nonmonotone filter Diagonalized Quasi-Newton Multiplier (DQMM) method for solving system of nonlinear equations. The system of nonlinear equations is transformed into a constrained nonlinear programming problem which is then solved by nonmonotone filter DQMM method. A nonmonotone criterion is used to speed up the convergence progress in some ill-conditioned cases. Under reasonable conditions, we give the global convergence properties. The numerical experiments are reported to show the effectiveness of the proposed algorithm.

1. Introduction

In this paper, we consider the system of nonlinear equations:

$$c_i(x) = 0, \quad i = 1, 2, \ldots, m, \tag{1.1}$$

where $x \in \mathbb{R}^n$ and $c_i(x) : \mathbb{R}^n \to \mathbb{R}$ for $i = 1, 2, \ldots, m$.

An important method is based on successive linearization, in which d_k is calculated on iteration k that solves the system of linear equations

$$c_i(x_k) + a_i(x_k)^T d_k = 0, \quad i = 1, 2, \ldots, m, \tag{1.2}$$

where $a_i(x_k) = \nabla c_i(x_k)$, $i = 1, 2, \ldots, m$. If $m = n$, (1.2) is Newton's method. But in some cases (1.2) is inconsistent.

An alternative approach is to pose (1.1) as a minimization problem

$$\min h(x) = \frac{1}{2}\|c(x)\|^2. \tag{1.3}$$

This idea helps to improve the global properties of Newton's method. But there are still potential difficulties. Powell [17] gives an example that the iterates converges to a non-stationary point of $h(x)$, which is obviously unsatisfactory.

The third strategy is recently proposed in [10,13,14]. In [10], Nie divides the set $\{1,2,\ldots,m\}$ into S_1 and S_2, where S_2 denotes the complement $\{1,2,\ldots,m\}/S_1$. The system of nonlinear equations is transformed into a nonlinear optimization

$$\min \quad \sum_{i \in S_1} c_i^2(x), \tag{1.4a}$$

$$\text{subject to } c_j(x) = 0, \quad j \in S_2, \tag{1.4b}$$

which is solved by a null space algorithm with trust region method. The penalty function in Nie's paper is used to guarantee the global convergence of algorithm, but there is much difficulty in choosing the penalty parameter in penalty function.

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Filter methods are recently presented by Fletcher and Leyffer [2] for nonlinear programming (NLP), offering an alternative to penalty functions, as a tool to guarantee global convergence of algorithms for nonlinear programming. The underlying concept is that trial points are accepted if they improve the objective function or improve the constraint violation. Filter methods have several advantages over penalty function methods. Firstly, no penalty parameter estimates, which could be difficult to obtain, are required. Secondly, filter approaches play an important role to balance the objective function and constraints. Therefore, this topic got high importance in recent years (see [3,4,6,7,9,12,15,16,18,19,21,22]).

Recent research indicates that the monotone line search technique may have some drawbacks. In particular, enforcing monotonicity may considerably reduce the rate of convergence when the iteration is trapped near a narrow curved valley. Grippo et al. [5] generalized the Armijo rule and proposed a nonmonotone line search technique for unrestricted optimization. Several numerical tests show that the nonmonotone line search technique is efficient and competitive.

Diagonalized Quasi-Newton Multiplier (DQMM) method as defined in Tapia [20] is employed to handle (1.4). The DQMM method is actually equivalent to the SQP method which is one of the most successful methods for equality constrained optimization. Motivated by the above ideas, we propose a nonmonotone filter DQMM method for system of nonlinear equations. The line search filter method is employed to guarantee the global convergence. A nonmonotone criterion is used to speed up the convergence progress in some ill-conditioned cases.

The paper is outlined as follows. In Section 2, we state the nonmonotone filter DQMM method; the global convergence of the algorithm is proved in Section 3; finally, we report some numerical experiments in Section 4.

Notation. $\|\cdot\|$ is the ordinary Euclidean norm in the paper.

2. Algorithm

The KKT conditions for the problem (1.4) are

$$g(x) + A_{S_2}(x)\lambda = 0, \tag{2.1a}$$

$$c_{S_2}(x) = 0, \tag{2.1b}$$

where $g(x) = \nabla\sum_{i\in S_1} c_i^2(x)$ denote the gradient of the function $\sum_{i\in S_1} c_i^2(x)$ and $A_{S_2} = [\nabla c_{S_2}(x)]^T$ denote the Jacobian of the constraint $c_{S_2}(x)$. In each iteration k a search direction s_k is computed from the linearization at x_k of the KKT condition,

$$\begin{pmatrix} H_k & A_{S_2}^k \\ (A_{S_2}^k)^T & O \end{pmatrix}\begin{pmatrix} s_k \\ \lambda_k^+ \end{pmatrix} = -\begin{pmatrix} g_k \\ c_{S_2}^k \end{pmatrix}, \tag{2.2}$$

where λ_k^+ can be used to determine a new estimate λ_{k+1} for the next iteration, the H_k denotes the Hessian $\nabla_{xx}^2\mathcal{L}(x_k,\lambda_k)$ of the Lagrangian function

$$\mathcal{L}(x,\lambda) = \sum_{i\in S_1} c_i^2(x) + \lambda^T c_{S_2}(x), \tag{2.3}$$

or an approximation to this Hessian. In order to obtain the KKT point of (2.1), we use the DQMM method which is given by the following iterative scheme.

$$\lambda_k = -\left(\left(A_{S_2}^k\right)^T B_k^{-1} A_{S_2}^k\right)^{-1}\left[c_{S_2}^k - \left(A_{S_2}^k\right)^T B_k^{-1} g_k\right], \tag{2.4}$$

$$B_k s_k = -\nabla_x\mathcal{L}(x_k,\lambda_{k+1}), \tag{2.5}$$

$$y_k = \nabla_x\mathcal{L}(x_k+s_k,\lambda_{k+1}) - \nabla_x\mathcal{L}(x_k,\lambda_{k+1}), \tag{2.6}$$

$$B_{k+1} = \mathrm{DFP/BFGS}(s_k, y_k, B_k), \tag{2.7}$$

$$x_{k+1} = x_k + s_k. \tag{2.8}$$

After a search direction s_k has been computed by (2.5), a step size $\alpha_k \in (0,1]$ is determined in order to obtain the next iterate

$$x_{k+1} = x_k + \alpha_k s_k.$$

The basic idea of filter method is to interpret the optimization problem as a biobjective optimization problem. In this article, we define two merit functions as follow

$$\psi(x) = \sum_{i\in S_1} c_i^2(x), \quad \theta(x) = \|c_{S_2}(x)\|^2.$$

A trial point $x_k(\alpha_{k,l}) = x_k + \alpha_{k,l}s_k$ is acceptable if

$$\psi(x_k(\alpha_{k,l})) \leqslant \max_{0\leqslant j\leqslant m(k)-1} \psi_{k-j} - \gamma_\psi\theta_k, \tag{2.9a}$$

or

$$\theta(x_k(\alpha_{k,l})) \leqslant (1-\gamma_\theta)\max_{0\leqslant j\leqslant m(k)-1} \theta_{k-j} \tag{2.9b}$$