Applied Mathematics and Computation 217 (2011) 9397–9403

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Exact solution of four cracks originating from an elliptical hole in one-dimensional hexagonal quasicrystals

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A R T I C L E I N F O

Keywords:
Quasicrystal
Crack
Hole
Stress intensity factor
Stroh-type formalism
Mapping function

A B S T R A C T

Based on the Stroh-type formalism for anti-plane deformation, the fracture mechanics of four cracks originating from an elliptical hole in a one-dimensional hexagonal quasicrystal are investigated under remotely uniform anti-plane shear loadings. The boundary value problem is reduced to Cauchy integral equations by a new mapping function, which is further solved analytically. The exact solutions in closed-form of the stress intensity factors for mode III crack problem are obtained. In the limiting cases, the well known results can be obtained from the present solutions. Moreover, new exact solutions for some complicated defects including three edge cracks originating from an elliptical hole, a half-plane with an edge crack originating from a half-elliptical hole, a half-plane with an edge crack originating from a half-circular hole are derived. In the absence of the phason field, the obtainable results in this paper match with the classical ones.

1. Introduction

Quasicrystal (QC) as a new structure and material was first observed by Shechtman et al. and announced in 1984 [1]. The theoretical frame of QCs comes from physical research which has been done by some researchers [2,3]. Experiments have shown that QCs are quite brittle [4] and the defects of quasicrystalline materials have been observed [5]. It is well known that the presence of defects such as holes and cracks, greatly affects the physical and mechanical properties of solid materials including QCs. Therefore, the study of crack problem of quasicrystalline materials is meaningful both in theoretical and practical applications.

Various mathematical methods developed by some investigators to obtain the exact analytical solutions for the crack problems of QCs. Fan [6] presented the mathematical theory of quasicrystalline elasticity. Using this theory, a straight dislocation and a moving screw dislocation in one-dimensional (1D) hexagonal QCs were addressed by Li and his coauthors [7,8]. Using Fourier series and Hankel transform methods, Peng and Fan [9] solved the crack and indentation problems of 1D hexagonal QCs. The perturbation method [10] was employed to solve the elastic problems of icosahedral quasicrystals containing a circular crack. Liu et al. [11] considered the interaction of between dislocations and cracks in 1D hexagonal QCs by the complex variable function method. Very recently, by developing the technique of conformal mapping, Guo and Liu [12–14] addressed the analytical solutions of several complicated defects such as cracks originating from holes in 1D hexagonal QCs. Li and Fan [15] obtained the exact solutions of two semi-infinite collinear cracks in a strip of 1D hexagonal QCs by introducing a conformal transformation. Wang and Pan [16] studied some typical defect problems in octagonal QCs and derived the general solutions of elastic fields by means of the differential operator theory and the complex variable method. Reducing the boundary value problem to the Riemann–Hilbert problem of periodic analytic functions, Shi [17]

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0096-3003/$ - see front matter © 2011 Elsevier Inc. All rights reserved.
doi:10.1016/j.amc.2011.04.028

obtained the closed-form solutions of collinear periodic cracks and/or rigid inclusions of antiplane sliding mode in 1D hexagonal QCs. From the viewpoint of the engineering application, the problem of cracks originating from holes is of particular interest. Because the stress concentrated effect around the hole easily induces crack occurrence and propagation when the holed structures of quasicrystalline materials are subjected to mechanical loadings. By using the technique of conformal mapping and the complex variable method, Guo and Liu [12–14] addressed the exact analytical solutions of two cracks originating from circular and elliptical holes in 1D hexagonal QCs.

Different from the above-mentioned methods, in this work we develop a Stroh-type formulism for anti-plane deformation in 1D hexagonal QCs and investigate the fracture mechanics of four cracks originating from an elliptical hole in a 1D hexagonal QC under uniform remote anti-plane shear loadings of the phonon field and the phason field. Then the boundary value problem is reduced to a Cauchy integral equation by a new mapping function, which is further solved analytically. The exact solutions of the stress intensity factors for the phonon field and the phason field are obtained respectively, which are very useful in practice.

2. Basic equations

When defects parallel to the quasi-periodic axis of 1D hexagonal QCs exist, the geometrical properties of the materials will be invariable along the quasi-periodic direction. In this case, the corresponding elasticity problem can be decomposed into two independent problems, i.e., a plane elasticity of conventional hexagonal crystal which can be solved by the route of the linear elastic theory [18] and an anti-plane phonon–phason field coupling elasticity problem [6]. Thus, we only need consider the latter one. The physical problem considered in this paper is shown in Fig. 1.

It is assumed that the quasi-periodic direction of 1D hexagonal QCs is along the positive direction of x_3 axis. In this case, all field variables are independent of x_3 and we have the following deformation geometrical equations [6]

$$\varepsilon_{3j} = \varepsilon_{j3} = u_{3,j}/2, \quad w_{3j} = v_{3,j}, \tag{1}$$

the equilibrium equations (if the body force is neglected)

$$\sigma_{3j,j} = 0, \quad H_{3j,j} = 0 \tag{2}$$

and the generalized Hooke's law

$$\sigma_{3j} = C_{44}u_{3,j} + R_3 v_{3,j}, \quad H_{3j} = R_3 u_{3j} + K_2 v_{3,j}, \tag{3}$$

where $j = 1, 2$; the repeated indices denote summation; a comma in the subscripts stands for a partial differentiation; σ_{ij}, ε_{3j}, u_3 are the stress, strain and displacement of the phonon field, respectively; H_{ij}, w_{ij}, v_3 are the stress, strain and displacement of the phason field; C_{44} and K_2 are the elastic constants of the phonon field and the phason field, respectively; R_3 is the phonon–phason coupling elastic constant.

Substituting Eq. (3) into Eq. (2) results in

$$\mathbf{B}_0\nabla^2\mathbf{u} = \mathbf{0}, \tag{4}$$

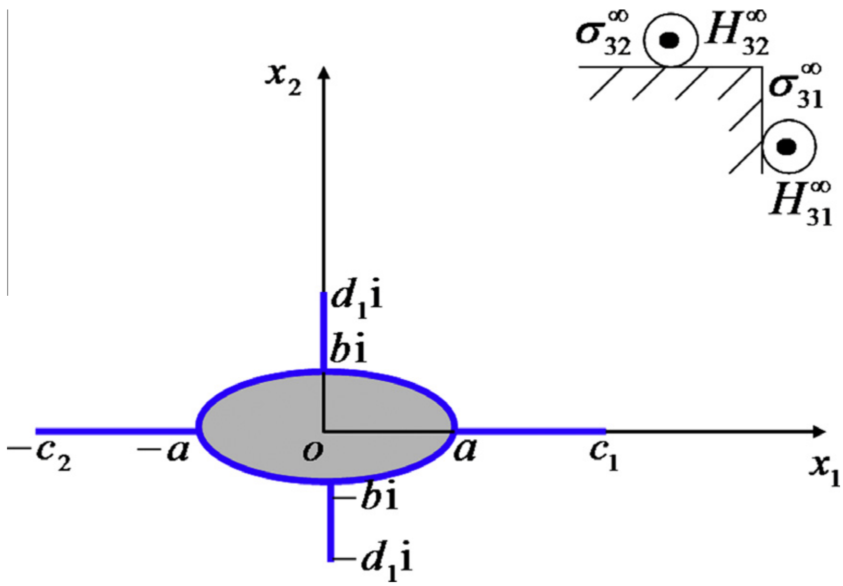

Fig. 1. Four cracks originating from an elliptical hole embedded in 1D hexagonal QCs.