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Impulsive harvesting and by-catch mortality for the theta logistic model

Tingting Zhao [a], Sanyi Tang [b,*]

[a] Department of Applied Mathematics, Xi'an Jiaotong University, Xi'an 710049, PR China
[b] College of Mathematics and Information Science, Shaanxi Normal University, Xi'an 710062, PR China

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ABSTRACT

In this paper, we investigate the population dynamics described by the theta logistic model with periodic impulsive harvesting and by-catch mortality. We examine the existence and stability of two positive periodic solutions by using qualitative methods and cobwebs. Then the sufficient conditions under which the unique positive periodic solution exists and is semi-stable are established, and qualifications for the solutions approach zero are also obtained. Further, choosing the maximum sustainable yield as the management objective, we investigate the optimal harvesting policy for the theta logistic model with periodic impulsive harvesting. Moreover the corresponding theta logistic difference equation is considered subject to the impulsive perturbation, and the dynamics which is parallel to that for the differential equation is examined. The main results extend and generalize the classical results for populations described by the autonomous logistic equation in renewable resources management.

1. Introduction

The sustainable development of renewable resources has been paid much attention by researchers for a long time [1–3]. Generally speaking, sustainable exploitation of renewable resources means one can get a relatively high level of productivity as well as keep the resources from going to extinction. Therefore, it is important to evolve management policies that maximize biomass yield and avoid over-exploitation so as to maintain sustainable development.

Many researchers formulated mathematical models to investigate the optimal management of renewable resources. Suppose that $N(t)$, the population size at time t, satisfies the well known logistic equation

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right), \tag{1}$$

where r, as a positive constant, is called the intrinsic growth rate, and the positive constant K is the environment carrying capacity, or saturation lever. Assume that the population described by (1) is subject to harvesting at a rate $h(N(t))$. Then the model for the exploited resources reads

$$\frac{dN}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) - h(N(t)). \tag{2}$$

The function $h(N(t))$ could be a constant or the catch-per-unit-effort hypothesis $h(N(t)) = EN(t)$, where E ($E \geqslant 0$) denotes the harvesting effort. When the harvesting is linear dependent on the density of the population, i.e. $h(N(t)) = EN(t)$, Fan [4] considered the optimal harvesting problem for (2) with periodic coefficients and generalized the classical results of Clark, who had considered the optimal harvesting policy for renewable resources [5].

* Corresponding author.
E-mail addresses: sanyitang219@hotmail.com, sytang@snnu.edu.cn (S. Tang).

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A common assumption for (2) is that the human exploitable activities occur continuously. The harvesting of species, however, is seasonal or it occurs in regular pulses [6]. So impulsive harvesting which provides a natural description of such situation is more realistic than the continuous strategy. Some impulsive equations have been introduced in population ecology [7–9] and chemotherapeutic treatment of disease [10]. Many other examples were given in Bainov's and his collaborators' book [11]. In particular, the following logistic model with impulsive harvest at fixed moments has been extensively investigated [3,12–14]

$$\begin{cases} \frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right), & t \neq \tau_k, \quad k = 1,2,\ldots, \\ N(\tau_k^+) = (1-E)N(\tau_k), & t = \tau_k, \quad k = 1,2,\ldots, \end{cases} \tag{3}$$

where the sequence $\{\tau_k\}$, the impulsive moments, satisfies $\tau_1 < \tau_2 < \cdots$ and $\lim_{k\to\infty}\tau_k = +\infty$, and for model (3) a natural assumption on the harvesting effort E is that $0 \leqslant E < 1$. The existence and stability of periodic solution of the model (3) were studied, and when the maximum biomass yield was chose as the management objective the optimal harvesting policy for the population was also investigated. Supposing the population is modelled by the Gompertz equation, Braverman and Mamdani [15] considered the similar issues with certain deduction was incorporated into the model in the impulsive moments and obtained the existence and stability of the impulsive periodic solutions.

Assume in present paper that the population is growing according to the following theta logistic growth equation [16]

$$\frac{dN(t)}{dt} = rN(t)\left[1 - \left(\frac{N(t)}{K}\right)^{\theta}\right], \tag{4}$$

where θ is a positive constant.

The theta logistic model (4) is a generalization of the standard logistic equation. This model firstly proposed by Gilpin and Ayala [16] is used for diverse purposes, including modeling tree growth, the growth of juvenile mammals and birds and so on [17–19]. It has an additional parameter θ, which is a shape parameter that can change the model (4) into the logistic or Gompertz equation by changing values of θ. In fact, as θ tends to zero the theta logistic model becomes to the Gompertz model [15]. For logistic model, the growth rate $rN(1 - \frac{N}{K})$ is a parabola that intersects the density axis at 0 and K and is symmetrical about $\frac{K}{2}$. But the parameter θ in (4) alters this restriction of symmetry. Further, data-fitting with the theta logistic model is very likely to simulate some experimental data. So the theta logistic model has been widely used instead of other sigmoid equations by reason of its availability as one of the standard equations offered for data-fitting by statistical software package [20].

Here, we connect proportional harvest with constant deduction named by-catch mortality, where the constant is denoted by D. Sometimes D is referred to as the parameter of constant harvesting, but in this paper it is assumed to be some constant harm (due to noise, pollution, change of mating, etc.). Furthermore, assume $\tau_{i+1} - \tau_i = T$ for all $i = 1,2,\ldots$, then the theta logistic equation with periodic impulsive harvest and by-catch mortality follows

$$\begin{cases} \frac{dN(t)}{dt} = rN(t)\left[1 - \left(\frac{N(t)}{K}\right)^{\theta}\right], & t \neq nT, \quad n = 1,2,\ldots, \\ N((nT)^+) = (1-E)N(nT) - D, & t = nT, \quad n = 1,2,\ldots, \\ N(0^+) = N(0) \doteq N_0, \end{cases} \tag{5}$$

where T is a fixed positive constant and denotes the period of the impulsive effects, and $D \geqslant 0, N_0 > 0$.

The organization of this paper is as follows: to begin with, we give some results which provide sufficient conditions of (5) for sustainable harvesting is possible and the population to be extinct, respectively. Meanwhile the stability of positive periodic solutions is studied. In subsequent portion of this paper, we focus on the maximum sustainable yield (*MSY*) and obtain analogous results for the corresponding difference equation with impulsive harvest and by-catch mortality. Finally some biological conclusions are discussed.

2. Existence and stability of periodic solutions of the system (5)

The main purposes of this section are to investigate the existence and stability of periodic solutions of Eq. (5), and further some sufficient conditions under which the population goes to extinction are obtained. Before giving the key results, we need the following definitions.

Definition 2.1 [15]. We will say a population goes to extinction if either there exists $t_0 > 0$ such that $N(t) = 0$ for $t \geqslant t_0$, or $N(t) > 0$ but $\liminf_{t\to\infty} N(t) = 0$, where $N(t)$ is the population size at time t.

Definition 2.2 [21]. If (5) has an asymptotically stable T-periodic solution $N(t)$, then the yield (per unit time)

$$Y = \frac{E}{T}N(nT) \tag{6}$$

is said to be a sustainable yield. If Y attains its maximum, then we have the maximum sustainable yield (*MSY*) (per unit time).