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Effect of viscous dissipation on natural convection heat and mass transfer from vertical cone in a non-Newtonian fluid saturated non-Darcy porous medium

R.R. Kairi *, P.V.S.N. Murthy

Department of Mathematics, Indian Institute of Technology, Kharagpur 721 302, India

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ABSTRACT

In this paper we investigate the influence of viscous dissipation and Soret effect on natural convection heat and mass transfer from vertical cone in a non-Darcy porous media saturated with non-Newtonian fluid. The surface of the cone and the ambient medium are maintained at constant but different levels of temperature and concentration. The Ostwald–de Waele power law model is used to characterize the non-Newtonian fluid behavior. The governing equations are non-dimensionalized into non-similar form and then solved numerically by local non-similarity method. The effect of non-Darcy parameter, viscous dissipation parameter, Soret parameter, buoyancy ratio, Lewis number and the power-law index parameter on the temperature and concentration field as well as on the heat and mass transfer coefficients is analyzed.

1. Introduction

The study of convection heat and mass transfer in a porous medium from axisymmetric body has attracted many investigators due to its wide range of applications in geophysics and energy related problems such as thermal insulation, enhanced recovery of petroleum resource, geophysical flows, polymer processing in packed beds and sensible heat storage bed. In order to design a suitable canister for nuclear waste disposal into the depth of the earth or into the sea bed demands a through understanding of the convective mechanism in a porous medium for taking care of the safety measure for all sea living beings. In this direction, one needs to study the convective heat and mass transfer from different geometries. To begin with, axisymmetric bodies such as a cone, horizontal and vertical cylinder, and sphere are used to understand convective heat and mass transfer mechanism.

In particular, a number of industrially important fluids including fossil fuels exhibit non-Newtonian fluid behavior. Non-Newtonian power law fluids are so widespread in industrial processes and in the environment that it would be no exaggeration to affirm that Newtonian shear flows are the exception rather than the rule. Shenoy [1] presented many interesting applications of non-Newtonian power law fluids with yield stress on convective heat transport in fluid saturated porous media considering geothermal and oil reservoir engineering applications. Natural convection from a vertical wall and that around a horizontal cylinder and a sphere in a non-Newtonian fluid saturated porous medium was presented by Chen and Chen in [2,3], respectively. Nakayama and Koyama [4] analyzed the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Non-Darcy natural, forced and mixed convection heat transfer in non-Newtonian power-law fluid saturated porous media was studied by Shenoy [5]. The uniform lateral mass flux

* Corresponding author.
E-mail addresses: rishi@maths.iitkgp.ernet.in (R.R. Kairi), pvsnm@maths.iitkgp.ernet.in (P.V.S.N. Murthy).

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Nomenclature

b	coefficient in the Forchheimer term
c_p	specific heat at constant pressure
C	concentration
d	pore diameter
D	solutal diffusivity
D_1	diffusion of mass due to temperature gradient
f	dimensionless stream function
g	acceleration due to gravity
K	permeability of the porous medium
K^*	modified permeability of the porous medium of power law fluid
Gr^*	non-Darcian (inertia) parameter or Grashof number based on permeability for power law fluid
n	power law index
N	buoyancy ratio
Nu	Nusselt number
r	local radius of the cone
Sh	Sherwood number
S_r	Soret number
Le	Lewis number
Ra_x	local Rayleigh number
T	temperature
x, y	axial and normal co-ordinates
u, v	velocity components in x and y directions

Greek symbols

μ^*	fluid consistency of the inelastic non-Newtonian power-law fluid
ρ	density
α	effective thermal diffusivity
ε	dissipation parameter
φ	porosity of the saturated porous medium
β_T	coefficient of thermal expansion
β_c	coefficient of solutal expansion
γ	half angle of the cone
ψ	dimensionless stream function
η	similarity variable
θ	dimensionless temperature
ϕ	dimensionless concentration
θ_w	$T_w - T_\infty$.
ϕ_w	$C_w - C_\infty$

Subscripts

w, ∞	conditions on the wall and the ambient medium

effect on natural convection of non-Newtonian fluids over a vertical cone in a porous media was investigated by Yih [6]. Natural convection heat and mass transfer from a vertical truncated cone in a porous medium saturated with a non-Newtonian fluid with variable wall temperature and concentration was reported by Cheng [7]. Similarity solution for the natural convection heat transfer of non-Newtonian fluids in porous media from a vertical cone under mixed thermal boundary conditions was obtained by Cheng [8].

Viscous dissipation acts as a heat source and generates appreciable temperature in the medium. In the porous medium, it is interpreted as the rate at which mechanical energy is converted into heat in a viscous fluid per unit volume (Bejan [9]). Viscous dissipation is of significance in natural convection in various devices that are subjected to large variations of gravitational force or that operate at high rotational speeds (Gebhart [10]). Gebhart and Mollendorf [11] analyzed the effect of viscous dissipation in external natural convection considering exponential variation of wall temperature through a similarity solution. A comment was made by Fand and Brucker [12] that the effect of viscous dissipation might become significant in case of natural convection in porous medium in connection with their experimental correlation for the heat transfer in external flows. The validity of the comment was tested for the Darcy model by Fand et al. [13], both experimentally and analytically while estimating the heat transfer coefficient from a horizontal cylinder embedded in a porous medium. Their mathematical analysis is confined to studying the dissipation effect using a steady, 1-D energy equation, on the basis of the equation form analogy given by Bejan [9] for the inclusion of viscous dissipation effects. Nakayama and Pop [14]