Applied Mathematics and Computation 217 (2011) 6792–6801

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Analysis of blood flow through a modelled artery with an aneurysm

S. Mukhopadhyay, G.C. Layek *

Department of Mathematics, The University of Burdwan, Burdwan 713 104, WB, India

ARTICLE INFO

Keywords:
Aneurysm
Haematocrit
Non-homogeneous
Viscous incompressible fluid
Finite-difference method
Laminar flow conditions

ABSTRACT

The intention of the present work is to carry out a systematic analysis of flow features in a tube, modelled as artery, having a local aneurysm in presence of haematocrit. The arterial model is treated to be axi-symmetric and rigid. The blood, flowing through the modelled artery, is treated to be Newtonian and non-homogeneous. For a thorough quantitative analysis of the flow characteristics such as wall pressure, flow velocity, wall shear stress, the unsteady incompressible Navier–Stokes equations in cylindrical polar co-ordinates under the laminar flow conditions are solved by using the finite-difference method. Finally, the numerical illustrations presented in this paper provide an effective measure to estimate the combined influence of haematocrit and aneurysm on flow characteristics. It is found that the magnitude of wall shear stress and also the length of separation increase with increasing values of the haematocrit parameter. The length of flow separation increases but the peak value of wall shear stress decreases with the increasing length of aneurysm. The peak value of wall shear stress as well as the length of separation increases with the increasing height of the aneurysm.

1. Introduction

Of late, blood flow through a diseased artery has become quite interesting from theoretical, experimental and clinical point of view. Interest in the dynamics of blood flow through arteries has led to the publication of a number of papers on related problem of steady laminar flow through tubes. Flow through arteries becomes complicated due to formation of aneurysm, a balloon like dilatation, found on the walls of a vessel where it has been weakened. Aneurysms are usually seen in arteries such as cerebral, carotid, thoracic, renal, abdominal, iliac, femoral, bronchial etc. It grows gradually as time elapses and grows faster as it becomes larger. It triggers the thrombus formation, extent of which has a correlation with the rate of growth of aneurysm and also with the degradation of vessel wall [1]. Now the combination and interaction of thrombosis and aneurysm further complicate the haemodynamics in diseased arterial vessels. Furthermore, it has been observed that when a surgical intervention is performed on a particular aneurysm, other aneurysms in its vicinity, seem to become active. This suggests that patients with aneurysm may be susceptible to both aneurysm formation and rupture. Haemodynamic factors like mechanical stresses, acute blood pressures and wall pressures seem to play an important role in predicting growth and rupture of an aneurysm.

Numerous health complications caused by aneurysm have drawn a significant interest in the understanding of the cause of the disease in order to develop effective methods to treat it. The ability to describe the flow through vessels having aneurysm would provide the possibility of diagnosing the disease in earlier stages, even before the aneurysm becomes clinically relevant, and is necessary for surgical intervention. To the authors' knowledge, there are quite a good number of studies [2–9]. In most of these studies, the flowing blood is assumed to be Newtonian and homogeneous. The assumption of Newtonian

* Corresponding author.
E-mail addresses: swati_bumath@yahoo.co.in (S. Mukhopadhyay), goralayek@yahoo.com (G.C. Layek).

0096-3003/$ - see front matter © 2011 Published by Elsevier Inc.
doi:10.1016/j.amc.2010.10.011

behaviour of blood is acceptable for high shear rate flow i.e. in the case of flow through larger arteries [10]. It is not, however, valid when the shear rate is low and in the case of flow through smaller arteries. Blood is composed of fluid plasma and formed elements. The formed elements of blood are erythrocytes, leukocytes and platelets. The percentage volume of red cells is called the haematocrit and is approximately 40–45% [11] for an adult. Red cells may affect the viscosity of whole blood and thus the velocity distribution depends on the concentration of cells. So blood can not be considered as homogeneous fluid [12]. For medical scientists, an accurate knowledge of the mechanical properties of whole blood and the erythrocytes can suggest a new diagnostic tool. For specialists in fluid mechanics, detailed informations of the complex rheological behaviour of the system is of utmost importance in any attempt towards establishing the equations that govern the flow of blood in the circulatory system. However, there is a lack of research in the area of modelling of blood flows in tube with aneurysm in presence of haematocrit.

Keeping the above motivation in mind, an attempt is made to explore the effects of aneurysm on the flow characteristics of blood taking into account that the flowing blood is to be treated Newtonian. The functional dependence of blood viscosity on haematocrit has been duly accounted for in order to improve resemblance to the real situation. In this paper, a numerical code (two-stage scheme in cylindrical polar co-ordinate system) has been used for the calculation of flow variables in the vicinity of an aneurysm. Marker and Cell (MAC) method [13] has been used to solve the governing equations of motion. The wall of arterial segment is considered to be rigid. In case of a diseased artery, the vessel walls in the vicinity of the affected part are usually relatively solid. Due attention has been paid to the physical aspects of the flow features in the vicinity of the aneurysm. The flow characteristics e.g. flow separation, pressure drop and arterial wall shear stress are also discussed at length through their graphical representations.

2. Model for blood viscosity

Lih [14] postulated a model for the non-uniform suspension viscosity for blood, flowing through an arterial tube having uniform cross section e.g.

$$\mu^*(r^*) = \mu_0^*\left[1 + k\left\{1 - \left(\frac{r^*}{a_0}\right)^n\right\}\right] \tag{1}$$

where $k = Bh_m$, B is a constant having the value 2.5 and h_m the maximum haematocrit at the center. The viscosity (μ_0^*) of the plasma is assumed constant. The power n is a parameter determining the shape of the cell distribution in blood flow. a_0 is the radius of the straight portion of the tube. In the present analysis, we consider three cases viz. $n = 1, 2, 3$. Formula (1) indicates that viscosity increases as we move from the wall towards the center where it is maximum. Also, it may be imagined that near the wall there is a thin layer dominated by plasma.

3. Equations of motion

The viscous, incompressible flow in a long tube with aneurysm at the specified position is considered. Let $(r*, \theta*, z*)$ be the cylindrical polar co-ordinates with $z*$-axis along the axis of symmetry of the tube. The region of interest is $0 \leqslant r* \leqslant r_0(z*)$, $0 \leqslant z* \leqslant L*$($L*$ being the finite length). Geometrical configuration of the tube is shown in Fig. 1. Let $u*$ and $v*$ be the axial and the radial velocity components respectively, $p*$ the pressure, ρ the density and v denotes the kinematic viscosity of the fluid. The fluid is assumed to be non-homogeneous and incompressible, and that the flow is also laminar. Blood in physiological conditions may be considered as incompressible [15]. We introduce the non-dimensional variables $t = t^*U/a_0,\ r = r^*/a_0, z = z^*/a_0,\ r_0(z) = r_0^*(z^*/a_0)/a_0,\ u = u^*/U,\ v = v^*/U,\ p = p^*/\rho U^2$ where a_0 is the radius of the straight

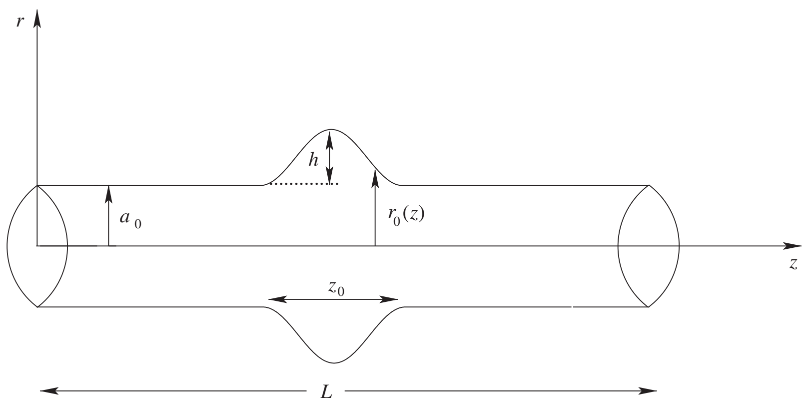

Fig. 1. Geometry of the tube with aneurysm.