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Controllability of impulsive differential systems with nonlocal conditions

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ABSTRACT

The paper is concerned with the controllability of impulsive functional differential equations with nonlocal conditions. Using the measure of noncompactness and Mönch fixed-point theorem, we establish some sufficient conditions for controllability. Firstly, we require the equicontinuity of evolution system, and next we only suppose that the evolution system is strongly continuous. Since we do not assume that the evolution system generates a compact semigroup, our theorems extend some analogous results of (impulsive) control systems.

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1. Introduction

In this paper, we consider the following impulsive functional differential systems:

$$x'(t) = A(t)x(t) + f(t, x(t)) + (Bu)(t), \text{ a.e. on } [0, b], \tag{1.1}$$

$$\Delta x(t_i) = x(t_i^+) - x(t_i^-) = I_i(x(t_i)), i = 1, \ldots, s, \tag{1.2}$$

$$x(0) + M(x) = x_0, \tag{1.3}$$

where $A(t)$ is a family of linear operators which generates an evolution operator

$$U : \triangle = \{(t, s) \in [0, b] \times [0, b] : 0 \leqslant s \leqslant t \leqslant b\} \to L(X),$$

here, X is a Banach space, $L(X)$ is the space of all bounded linear operators in X; $f : [0,b] \times X \to X$; $0 < t_1 < \cdots < t_s < t_{s+1} = b$; $I_i : X \to X$, $i = 1,\ldots,s$ are impulsive functions; $M : PC([0,b];X) \to X$; B is a bounded linear operator from a Banach space V to X and the control function $u(\cdot)$ is given in $L^2([0,b],V)$.

Controllability for differential systems in Banach spaces has been studied by many authors [2,4,9] and the references therein. Benchohra and Ntouyas [4], using the Martelli fixed-point theorem, studied the controllability of second-order differential inclusions in Banach spaces. Guo et al. [9] proved the controllability of impulsive evolution inclusions with nonlocal conditions.

The impulsive differential systems can be used to model processes which are subjected to abrupt changes. The study of dynamical systems with impulsive effects has been an object of intensive investigations [8,14,15]. The semilinear nonlocal initial problem was first discussed by Byszewski [5,6] and the importance of the problem consists in the fact that it is more general and has better effect than the classical initial conditions. Therefore it has been studied extensively under various conditions on A(or $A(t)$) and f by several authors [1,11,13,17].

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Recently, Hernández and O'Regan [10] point out that some papers on exact controllability of abstract control system contain a similar technical error when the compactness of semigroup and other hypotheses are satisfied, that is, in this case the application of controllability results are restricted to finite dimensional space. The goal of this paper is to find conditions guaranteeing the controllability of impulsive differential systems when the Banach space is nonseparable and evolution systems $U(t,s)$ is not compact, by means of Mönch fixed-point theorem and the measure of noncompactness. Since the method used in this paper is also available for evolution inclusions in Banach space, we can improve the corresponding results in [2,4].

2. Preliminaries

Let $(X,\|\cdot\|)$ be a real Banach space. We denote by $C([0,b];X)$ the space of X-valued continuous functions on $[0,b]$ with the norm $\|x\| = \sup\{\|x(t)\|, t\in[0,b]\}$ and by $L^1([0,b];X)$ the space of X-valued Bochner integrable functions on $[0,b]$ with the norm $\|f\|_{L^1} = \int_0^b \|f(t)\|\,dt$.

For the sake of simplicity, we put $J = [0,b]$; $J_0 = [0,t_1]$; $J_i = (t_i, t_{i+1}]$, $i = 1,\dots,s$. In order to define the mild solution of problem (1.1)–(1.3), we introduce the set $PC([0,b];X) = \{u : [0,b] \to X : u$ is continuous on J_i, $i = 0,1,\dots,s$ and the right limit $u(t_i^+)$ exists, $i = 1,\dots,s\}$. It is easy to verify that $PC([0,b];X)$ is a Banach space with the norm $\|u\|_{PC} = \sup\{\|u(t)\|, t\in[0,b]\}$.

Let us recall the following definitions.

Definition 2.1. Let E^+ be the positive cone of an order Banach space $(E,\leqslant)$. A function Φ defined on the set of all bounded subsets of the Banach space X with values in E^+ is called a measure of noncompactness (MNC) on X if $\Phi(\overline{co}\Omega) = \Phi(\Omega)$ for all bounded subsets $\Omega\subset X$, where $\overline{co}\Omega$ stands for the closed convex hull of Ω.

The MNC Φ is said:

(1) *monotone* if for all bounded subsets Ω_1, Ω_2 of X we have: $(\Omega_1 \subseteq \Omega_2) \Rightarrow (\Phi(\Omega_1) \leqslant \Phi(\Omega_2))$;
(2) *nonsingular* if $\Phi(\{a\}\cup\Omega) = \Phi(\Omega)$ for every $a\in X$, $\Omega\subset X$;
(3) *regular* if $\Phi(\Omega) = 0$ if and only if Ω is relatively compact in X.
One of the most important examples of MNC is the noncompactness measure of Hausdorff β defined on each bounded subset Ω of X by

$$\beta(\Omega) = \inf\{\varepsilon > 0; \Omega \text{ has a finite } \varepsilon-\text{net in } X\}.$$

It is well known that MNC β enjoys the above properties and other properties (see [3,12]): for all bounded subset Ω, Ω_1, Ω_2 of X,

(4) $\beta(\Omega_1 + \Omega_2) \leqslant \beta(\Omega_1) + \beta(\Omega_2)$, where $\Omega_1 + \Omega_2 = \{x + y : x\in\Omega_1, y\in\Omega_2\}$;
(5) $\beta(\Omega_1\cup\Omega_2) \leqslant \max\{\beta(\Omega_1),\beta(\Omega_2)\}$;
(6) $\beta(\lambda\Omega) \leqslant |\lambda|\beta(\Omega)$ for any $\lambda\in R$;
(7) If the map $Q : D(Q) \subseteq X \to Z$ is Lipschitz continuous with constant k, then $\beta_Z(Q\Omega) \leqslant k\beta(\Omega)$ for any bounded subset $\Omega\subseteq D(Q)$, where Z is a Banach space.

Definition 2.2. A function $x(\cdot)\in PC([0,b];X)$ is a mild solution of (1.1)–(1.3) if

$$x(t) = U(t,0)x(0) + \int_0^t U(t,s)(f + Bu)(s)\,ds + \sum_{0<t_i<t} U(t,t_i)I_i(x(t_i)),$$

for all $t\in[0,b]$, where $x(0) + M(x) = x_0$.

Definition 2.3. The system (1.1)–(1.3) is said to be nonlocally controllable on J if, for every $x_0, x_1\in X$, there exists a control $u\in L^2(J,V)$ such that the mild solution $x(\cdot)$ of (1.1)–(1.3) satisfies $x(b) + M(x) = x_1$.

A two parameter family of bounded linear operators $U(t,s)$, $0\leqslant s\leqslant t\leqslant b$ on X is called an evolution system if the following two conditions are satisfied:

(i) $U(s,s) = I$, $U(t,r)U(r,s) = U(t,s)$ for $0\leqslant s\leqslant r\leqslant t\leqslant b$;
(ii) $(t,s)\to U(t,s)$ is strongly continuous for $0\leqslant s\leqslant t\leqslant b$.

Since the evolution system $U(t,s)$ is strongly continuous on the compact set $\triangle = J\times J$, then there exists $L_U > 0$ such that $\|U(t,s)\|\leqslant L_U$ for any $(t,s)\in\triangle$. More details about evolution system can be found in [18].

Definition 2.4. A countable set $\{f_n\}_{n=1}^{+\infty}\subset L^1([0,b];X)$ is said to be semicompact if:

- the sequence $\{f_n(t)\}_{n=1}^{+\infty}$ is relatively compact in X for a.a. $t\in[0,b]$;
- there is a function $\mu\in L^1([0,b];R^+)$ satisfying $\sup_{n\geqslant1}\|f_n(t)\|\leqslant\mu(t)$ for a.e. $t\in[0,b]$.

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