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Dynamic model of worms with vertical transmission in computer network

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ABSTRACT

An e-epidemic SEIRS model for the transmission of worms in computer network through vertical transmission is formulated. It has been observed that if the basic reproduction number is less than or equal to one, the infected part of the nodes disappear and the worm dies out, but if the basic reproduction number is greater than one, the infected nodes exists and the worms persist at an endemic equilibrium state. Numerical methods are employed to solve and simulate the system of equations developed. We have analyzed the behavior of the susceptible, exposed, infected and recovered nodes in the computer network with real parametric values.

1. Introduction

The advent of Internet/Network technology in the past three decades has led to sea change in the way data is transferred and information exchange takes place. Over the years coupled with technological development and need, Internet technology has grown, offering numerous functionalities and facilities. The growth of Internet technology has thrown severe challenges in form of requirement of a suitable cyber defense system to safeguard the valuable information stored on system and for information in transit. Towards this goal it makes us necessary to study and understand the different type of worms and develop mathematical models to represent their behavior. Worms behave like infectious diseases and are epidemic in nature. A computer worm is a self contained program that is able to spread functional copies of itself or its segment to other computer system without a dependency on another program to host its code. Model's ability to predict worm's behavior depends greatly on the assumptions made in the modeling process. The mathematical models will be generalized to represent the behavior of numerous other worms. The generalized model will be incorporated into a cyber defense system to proactively safeguard the information and information interchange. E-mail is one of the core media to exchange e-messages for better communication and connectivity but it is also a good media to send worms to the interconnected networks. By clicking incidentally or wrongly an attachment of malicious executable file the system may be infected, here the user's awareness is necessary to avoid such type of attacks.

The action of worms throughout a network can be studied by using epidemiological models for disease propagation [1–10]. Based on the Kermack and McKendrick SIR classical epidemic model [11–13], dynamical models for malicious objects propagation were proposed, providing estimations for temporal evolutions of nodes depending on network parameters considering topological aspects of the network [1–3,14–17]. The kind of approach was applied to e-mail propagation schemes [18] and modification of SIR models generated guides for infection prevention by using the concept of epidemiological threshold [1–3,19]. Richard and Mark [20] propose an improved SEI (susceptible-exposed-infected) model to simulate virus propagation. However, they do not show the length of latency and take into account the impact of anti-virus software.

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The model SEIR proposed by the authors [21] assumes that recovery hosts have a permanent immunization period with a certain probability, which is not consistent with real situation. In order to overcome limitation, Mishra and Saini [1] present an SEIRS model with latent and temporary immune periods, which can reveal common worm propagation. Recently, more research attention has been paid to the combination of virus propagation model and antivirus countermeasures to study the prevalence of virus, e.g., virus immunization [3,22–26,47–49] and quarantine [27–29]. Extending the SEIRS model of [1], Mishra et al. introduced new compartment quarantine and its effect has been analyzed in [30].

In the computer network some worms may pass from main server to any of the nodes via vertical transmission. The literature dealing with the analysis of worms that are both vertically and horizontally is not extensive. In the situation of effected nodes with vertical transmission, the anti-worms software is taken only to a proportion if new nodes that have not yet been infected. Few studies are written on this aspect. It has been observed that there is no permanent recovery from the worm till a node is attached to the computer network, even after the run of anti-worm software. We propose a SEIRS epidemic model with vertical transmission in which there is a constant period of temporary immunity (due to run of anti-worm software) of fixed length following temporary recovery from the infection of worm in place of an exponentially distributed period of temporary immunity.

2. Formulation of e-SEIRS epidemic worm model

A population size $N(t)$, i.e., the total nodes at any time t in the computer network, is partitioned into subclasses of nodes which are susceptible, exposed (infected but not yet infectious), infectious and recovered with sizes denoted by $S(t)$, $E(t)$, $I(t)$ and $R(t)$ *respectively.* i.e.,

$$S(t) + E(t) + I(t) + R(t) = N(t). \tag{1}$$

Our assumption on the dynamical transfer of the population is depicted in Fig. 1.

The SEIRS model with mass action incidence is,

$$\begin{aligned} S'(t) &= b - \lambda IS - pbE - qbI - dS + \zeta R, \\ E'(t) &= \lambda IS + pbE + qbI - \varepsilon E - dE, \\ I'(t) &= \varepsilon E - \gamma I - dI - \eta I, \\ R'(t) &= \gamma I - \zeta R - dR, \end{aligned} \tag{2}$$

where b, d, λ are positive constants and ε, η, γ, ζ are non-negative constants. The constants b is the recruitment rate of susceptible nodes to the computer network, d is the per capita natural mortality rate (i.e. the crashing of nodes due to the reason other than the attack of worms), ε is the rate constant for nodes leaving the exposed class E for infective class I, γ is the rate constant for nodes leaving the infective class I for recovered class R, η is the disease related death rate (i.e. crashing of nodes due to the attack of worms) in the class I, ζ is the rate constant for nodes becoming susceptible again after recovered.

In the SEIRS model, the flow is from class S to class E, class E to class I, class I to class R and again class R to class S. For the vertical transformation, we assume that a fraction p and a fraction q of the new nodes from the exposed and the infectious classes, respectively, are introduced into the exposed class E. Consequently, the birth flux into the exposed class is given by $pbE + qbI$ and the birth flux into the susceptible class is given by $b - pbE - qbI$.

3. Global stability of an e-SEIRS worm model

The system of Eq. (2) can be reduced to the equivalent system (by using Eq. (1) i.e. $R = N - S - E - I$)

$$\begin{aligned} S'(t) &= b - \lambda IS - pbE - qbI - dS + \zeta(N - S - E - I), \\ E'(t) &= \lambda IS + pbE + qbI - (\varepsilon + d)E, \\ I'(t) &= \varepsilon E - (\gamma + d + \eta)I, \end{aligned} \tag{3}$$

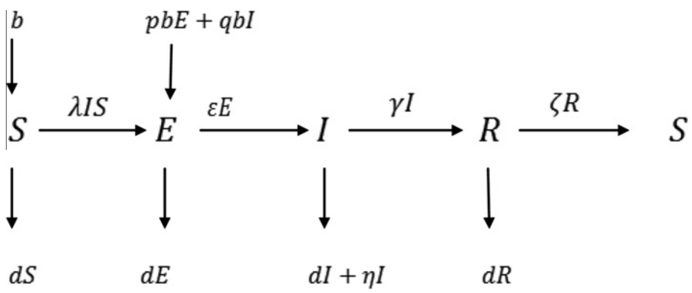

Fig. 1. Schematic diagram for the flow of worms in computer network.