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A new mathematical construction of the general nonlinear ODEs of motion in rigid body dynamics (Euler's equations)

Dimitrios E. Panayotounakos, Ioanna Rizou, Efstathios E. Theotokoglou *

School of Applied Mathematical and Physical Sciences (SEMFE), Department of Mechanics – Laboratory of Testing and Materials, National Technical University of Athens (NTUA), 5 Heroes of Polytechnium Avenue, Zographou, Athens GR 157-73, Hellas, Greece

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ABSTRACT

It is shown that the three nonlinear dynamic Euler ordinary differential equations (ODEs), concerning the motion of a rigid body free to rotate about a fixed point, are reduced, by means of a subsidiary function which is to be determined, to three Abel equations of the second kind of the normal form. Based on a recently developed mathematical construction concerning exact analytic solutions of the Abel nonlinear ODEs of the second kind, we perform a new mathematical solution for the classical dynamic Euler nonlinear ODEs.
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1. Introduction

There are many studies [1–4] on the motion of an asymmetric rigid body free to rotate about a fixed point. The so-called three dynamic and three kinematic Euler equations, governing the above mentioned problem, were mainly applied to self-excited gyros [5–7] and they were numerically or analytically solved for special cases of motion, depending on the kind of geometry of the body as well as the external loading. Successful attempts were made by Panayotounakos and Theocaris [8,9] in solving both systems of dynamic and kinematic Euler's equations by making use of several ad hoc assumptions concerning the kind of loading. Finally, during the forty last years several successive efforts were made by many researchers in analytically solving the so-called generalized Euler dynamic equations of motion [10–13]. However, the obtained solutions concern special symmetries and loadings.

This work deals with the possibility of constructing exact analytic solutions concerning the dynamic Euler equations of motion. We prove that the above mentioned Euler's nonlinear first-order differential system can be reduced, by means of a subsidiary function that is to be determined, to three nonlinear Abel equations of the second kind of the normal form. It is well-known that this type of equation admits exact analytic solutions in terms of known (tabulated) functions only for special cases depending on the form of their second free members [14]. Based on a recently developed mathematical construction [15,16] that performs exact analytic solutions of the previously mentioned Abel equation in the general case of an arbitrary smooth second member, we succeed in solving analytically the prescribed problem in the most general case of geometry and loading. Finally three applications have been made.

2. Preliminaries – Notation

Consider the motion of an arbitrary rigid body, free to rotate about a fixed point O. Let us denote by $Ox_1y_1z_1$ the space-fixed coordinate system, while by $Oxyz$ the body-fixed coordinate system. The state motion is a relation with an

* Corresponding author.
E-mail address: stathis@central.ntua.gr (E.E. Theotokoglou).

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instantaneous angular velocity $\omega(t) = (\omega_x, \omega_y, \omega_z)$ with respect to the *Oxyz*-system; *t* denotes the time. Let also J_x, J_y, J_z denote the principal moments of inertia of the body, corresponding to the *x*, *y*, *z*-axes, respectively, with an arbitrary sequence $J_x < J_y < J_z$. The body is then asymmetric and in the absence of friction the external forces reduce to a simple force **R** and a couple **M** through *O*. Thus, the vector **M** is known and the motion of the body can be obtained by the angular momentum theorem.

On the basis of the previous assumptions we may derive the well-known equations of motion of a rigid body free to rotate about a fixed point (Euler's dynamic equations of motion) in terms of the projections on the principal axes of inertia of the body through that point, namely:

$$J_x\omega'_{x_t} - a_1\omega_y\omega_z = M_x(t), \quad J_y\omega'_{y_t} - a_2\omega_x\omega_z = M_y(t), \quad J_z\omega'_{z_t} - a_3\omega_x\omega_y = M_z(t), \tag{1}$$

where

$$a_1 = J_y - J_z < 0, \quad a_2 = J_z - J_x > 0, \quad a_3 = J_x - J_y < 0, \tag{2}$$

while the symbol $y'_x = \frac{dy}{dx}, \ y''_{xx} = \frac{d^2y}{dx^2}$ is used for the total derivatives. Because the moment-vector **M** is known, the solution of the system Eq. (1) provides the resultants ω_x, ω_y, ω_z of the instantaneous angular velocity ω.

In what follows, based on a recently developed mathematical construction [15,16], we will construct a new mathematical solution of the dynamic equations of motion Eq. (1) in the general case when M_x, M_y and M_z are arbitrary smooth functions of time *t*.

3. A new mathematical construction

Let us now consider the Abel ODE of the first kind of the normal form

$$yy'_x - y = f(x), \tag{3}$$

where *f* is an arbitrary smooth function.

According to Refs. [15,16], the solution of this equation is given as follows:

$$\begin{aligned} y(x) &= \frac{1}{2}(x+2\lambda)\left[\overline{N}(x) + \frac{1}{3}\right], \\ \overline{N}'_x &= \frac{4(G+2f)}{(x+2\lambda)^2\left[\overline{N}(x)+\frac{4}{3}\right]}, \end{aligned} \tag{4}$$

where $G(x)$ is a subsidiary function being defined below, λ is a parameter and $\overline{N}(x)$ is one of the roots of the cubic equation of Cardano's form

$$\overline{N}^3(x) + p\overline{N}(x) + q = 0. \tag{5}$$

That is to say, $\overline{N}(x)$ is given by one of the following six functions:

Case a: $Q < 0$, $(p < 0)$

$$\overline{N}_1(x) = 2\sqrt{-\frac{p}{3}}\cos\frac{\phi}{3}, \quad \overline{N}_2(x) = -2\sqrt{-\frac{p}{3}}\cos\frac{\phi-\pi}{3}, \quad \overline{N}_3(x) = -2\sqrt{-\frac{p}{3}}\cos\frac{\phi+\pi}{3}, \tag{6}$$

where

$$\cos\phi = -\frac{q}{2\sqrt{-\left(\frac{p}{3}\right)^3}}, \quad 0 < \phi < \pi.$$

Case b: $Q > 0$

$$\overline{N}(x) = \sqrt[3]{-\frac{q}{2} + \sqrt{Q}} + \sqrt[3]{-\frac{q}{2} - \sqrt{Q}}. \tag{7}$$

Case c: $Q = 0$

$$\overline{N}_1(x) = 2\sqrt[3]{-\frac{q}{2}}, \quad \overline{N}_2(x) = \overline{N}_3(x) = -\sqrt[3]{-\frac{q}{2}}. \tag{8}$$

In these expressions $Q(x)$, $p(x)$ and $q(x)$ are given by

$$Q(x) = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2, \quad p = -\frac{a^2}{3} + b, \quad q = 2\left(\frac{a}{3}\right)^3 - \frac{ab}{3} + c, \tag{9}$$

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