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Plane strain deformation of an initially unstressed elastic medium

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ABSTRACT

Selim and Ahmed [1] used the eigenvalue approach by assuming distinct eigenvalues to calculate the elastic deformation due to an inclined load at any point as a result of an inclined line load of initially stressed orthotropic elastic medium. They studied the plane strain problem and obtained the corresponding results for an unstressed orthotropic medium as a particular case. In the present paper, it is shown that all the eigenvalues do not remain distinct, but become repeated when the elastic medium is free from the initial compressive stresses. Further, the displacements and stresses for an unstressed elastic medium have been independently obtained. The variation of the displacements and stresses due to normal and tangential line load are also shown graphically.

1. Introduction

The behavior of elastic materials due to loading is of great interest in engineering, soil mechanics and geophysics. When the source surface is very long in one direction in comparison to the others, the use of two-dimensional approximation is justified and consequently calculations are simplified to a great extent and one gets a closed-form analytical solution. A very long strip-source and a very long line source are examples of such two-dimensional sources. The deformation due to loading such as inclined line load, strip-load, continuous line load, etc., is useful in analyzing the field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation fields in the entire volume surrounding the source region.

Love [2] obtained expressions for the displacements due to line source in an isotropic elastic medium. Maruyama [3] obtained displacement and stress fields corresponding to long strike-slip faults in a homogenous isotropic half-space. Okada [4,5] provided compact analytical expressions for the surface deformation and internal deformation due to inclined shear and tensile faults in a homogenous isotropic half-space.

Using the body-force equivalent of dislocation source, as discussed by Burridge and Knopoff [6] and Aki and Richards [7], Pan [8] obtained the response of transversely isotropic layered medium to general dislocation sources. Garg et al. [9] obtained the representation of seismic sources causing anti-plane strain deformation of an orthotropic medium. Kumar et al. [10] used eigenvalue approach to solve the plane strain problem of poroelasticity for an isotropic medium. The corresponding problem for a transversely isotropic medium has been discussed by Kumar et al. [11].

Garg et al. [12] has studied the general plane strain problem of an infinite orthotropic elastic medium, due to two-dimensional sources, without considering the effect of the initial stress present in the medium. By considering distinct eigenvalues, they have used eigenvalue approach to obtain the deformation due to inclined line load. Selim and Ahmed [1] used the same technique to obtain analytical expressions for displacements and stresses at any point as a result of an inclined line load of initially stressed orthotropic elastic medium. It has also been discussed there that the corresponding deformation for an

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unstressed medium can be obtained as a particular case. In the present paper we have shown that all the eigenvalues do not remain distinct, rather become repeated when the elastic medium is free from the initial compressive stresses. Therefore, this case cannot be considered as a particular case of the results of Selim and Ahmed [1]. Here, we have obtained the displacements and stresses for an unstressed elastic medium by using the novel analytical eigenvalue method given by Ross [13] for repeated eigenvalues. The variation of the displacements and stresses due to normal and tangential line load are also shown graphically.

2. Theory

Selim and Ahmed [1] used an eigenvalue approach and obtained the analytical expressions for the displacement and stresses at any point due to a normal and tangential line loading at the origin of the xy-plane of an initially stressed orthotropic infinite elastic medium. They have considered the plane strain problem. For ready reference the expressions are reproduced hereunder:

2.1. Normal line load

$$u^N(x,y)=\frac{F_1}{4\pi B_{11}(P_2\xi_2-P_1\xi_1)}\times\left(P_1\log\left(y^2+\xi_1^2x^2\right)-P_2\log\left(y^2+\xi_2^2x^2\right)\right),$$

$$v^N(x,y)=\mp\frac{F_1}{4\pi B_{11}(P_2\xi_2-P_1\xi_1)}\times\left(\log\left(y^2+\xi_1^2x^2\right)-\log\left(y^2+\xi_2^2x^2\right)\right),$$

$$S_{11}^N(x,y)=\frac{xF_1}{2\pi(P_2\xi_2-P_1\xi_1)}\times\left(\frac{y^2\left(P_1\xi_1^2-P_2\xi_2^2\right)+\xi_1^2\xi_2^2(P_1-P_2)}{\left(y^2+\xi_1^2x^2\right)\left(y^2+\xi_2^2x^2\right)}\right)\pm\frac{B_{21}x^2yF_1\left(\xi_1^2-\xi_2^2\right)}{2\pi B_{11}(P_2\xi_2-P_1\xi_1)\left(y^2+\xi_1^2x^2\right)\left(y^2+\xi_2^2x^2\right)},$$

$$S_{12}^N(x,y)=\frac{yN_1F_1}{2\pi B_{11}(P_2\xi_2-P_1\xi_1)}\times\left(\frac{y^2(P_1-P_2)\mp xy\left(\xi_1^2-\xi_2^2\right)+x^2\left(P_1\xi_2^2-P_2\xi_1^2\right)}{\left(y^2+\xi_1^2x^2\right)\left(y^2+\xi_2^2x^2\right)}\right),\tag{1}$$

where the upper sign is for medium I and the lower sign is for medium II and superscript (N) indicates the results due to the normal line load F_1.

2.2. Tangential line load

$$u^T(x,y)=\pm\frac{F_2P_1P_2}{4\pi N_1(P_2\xi_1-P_1\xi_2)}\times\left(\log\left(y^2+\xi_1^2x^2\right)-\log\left(y^2+\xi_2^2x^2\right)\right),$$

$$v^T(x,y)=\frac{-F_2}{4\pi N_1(P_2\xi_1-P_1\xi_2)}\times\left(P_2\log\left(y^2+\xi_1^2x^2\right)-P_1\log\left(y^2+\xi_2^2x^2\right)\right),$$

$$S_{11}^T(x,y)=\frac{B_{21}yF_2}{2\pi N_1(P_2\xi_1-P_1\xi_2)}\times\left(\frac{y^2(P_1-P_2)+x^2\left(P_1\xi_1^2-P_2\xi_2^2\right)}{\left(y^2+\xi_1^2x^2\right)\left(y^2+\xi_2^2x^2\right)}\right)\pm\frac{B_{11}P_1P_2xy^2F_2\left(\xi_1^2-\xi_2^2\right)}{2\pi N_1(P_2\xi_2-P_1\xi_1)\left(y^2+\xi_1^2x^2\right)\left(y^2+\xi_2^2x^2\right)},$$

$$S_{12}^T(x,y)=\frac{xF_2}{2\pi(P_2\xi_1-P_1\xi_2)}\times\left(\frac{x^2\xi_1^2\xi_2^2(P_1-P_2)+y^2\left(P_1\xi_2^2-P_2\xi_1^2\right)}{\left(y^2+\xi_1^2x^2\right)\left(y^2+\xi_2^2x^2\right)}\right)+\frac{P_1P_2\left(\xi_2^2-\xi_1^2\right)x^2yF_2}{\left(y^2+\xi_1^2x^2\right)\left(y^2+\xi_2^2x^2\right)},\tag{2}$$

where (T) indicates the results due to tangential line load F_2 .

The stress–strain relations for initially unstressed orthotropic elastic medium for plane strain problem are Biot [14]

$$\begin{aligned}S_{11}&=B_{11}e_{11}+B_{12}e_{22},\\S_{22}&=(B_{12}-P)e_{11}+B_{22}e_{22},\\S_{12}&=2Q_3e_{12}.\end{aligned}\tag{3}$$

Here S_{ij} (i,j = 1,2) are the incremental stress components, B_{ij} and Q_3 are the incremental elastic coefficients and shear modulus respectively.

These incremental elastic coefficients are related to Lame's coefficients λ and μ of the isotropic unstressed state and are given by

$$\begin{aligned}&B_{11}=\lambda+2\mu(1+\zeta),\quad B_{12}=\lambda+2\mu\zeta,\\&B_{21}=\lambda,\quad B_{22}=\lambda+2\mu,\quad Q_3=\mu,\end{aligned}\tag{4}$$