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Minimizing total weighted completion time in a two-machine flow shop scheduling under simple linear deterioration

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ABSTRACT

In this paper, we address a two-machine flow shop scheduling problem under simple linear deterioration. By a simple linear deterioration function, we mean that the processing time of a job is a simple linear function of its execution start time. The objective is to find a sequence that minimizes total weighted completion time. Optimal schedules are obtained for some special cases. For the general case, several dominance properties and two lower bounds are derived to speed up the elimination process of a branch-and-bound algorithm. A heuristic algorithm is also proposed to overcome the inefficiency of the branch-and-bound algorithm. Computational analysis on randomly generated problems is conducted to evaluate the branch-and-bound algorithm and heuristic algorithm.

1. Introduction

There is a growing interest in the literature to study scheduling problems of *deteriorating jobs*, i.e., jobs whose processing times are increasing (decreasing) functions of their starting times. Such deterioration appears, e.g., in scheduling of learning activities, scheduling of maintaining jobs, modelling of fire fighting, cleaning assignments, etc. Extensive surveys of different models and problems concerning deteriorating jobs can be found in Alidaee and Womer [1], Cheng et al. [2], and Gawiejnowicz [3].

Most of these studies focus on single machine settings (see, e.g., Browne and Yechiali [4], Cheng and Ding [5], Ho et al. [6], Mosheiov [7,8], Sundararaghavan and Kunnathur [9], Wang and Xia [10], Cheng et al. [11], Kang and Ng [12], Wu et al. [13], Wang and Guo [14], Cheng et al. [15], Gawiejnowicz and Lin [16], and Yang [17]). Among the exceptions to the single machine studies. Chen [18], Mosheiov [19], Ji and Cheng [20], Kuo and Yang [21], and Huang and Wang considered scheduling deteriorating jobs in a *multi-machine* setting. They assumed linear deterioration and parallel identical machines. Chen [18] considered minimum flow time and Mosheiov [19] studied makespan minimization. Ji and Cheng [20] studied the parallel-machine scheduling problem with a simple linear deterioration to minimize the total completion time. They gave a fully polynomial-time approximation scheme for the case with m machines, where m is fixed. Kuo and Yang [21] considered a parallel-machine scheduling problem in which the processing time of a job is a linear function of its starting time. The objectives are to minimize the total completion of all jobs and the total load on all machines respectively. They showed that the problems are polynomially solvable. Huang and Wang [22] considered parallel identical machines scheduling problems with deteriorating jobs. The objectives are to minimize the total absolute differences in completion times (TADC) and the total absolute differences in waiting times (TADW). They showed that the problems remains polynomially solvable under the proposed model.

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Mosheiov [23] considered makespan minimization problem in flow shop, open shop and job shop with simple linear deteriorating jobs. He introduced a polynomial-time algorithm for the two-machine flow shop and proved NP-hardness when an arbitrary number of machines (three and above) is assumed. Wang and Xia [24] considered no-wait and no-idle flow shop scheduling problems with job processing times dependent on their starting times. In these problems the job processing time is a simple linear function of the job's starting time and some dominating relationships between machines are satisfied. They showed that for the problems to minimize makespan or minimize weighted sum of completion time, polynomial algorithms still exist. When the objective is to minimize maximum lateness, the solutions of a classical version may not hold. Wang et al. [25] considered a two-machine flow shop scheduling problem with a simple linear deterioration function. The objective was to find a sequence that minimizes total completion time. They proved that some special cases can be solved Optimally. They gave several dominance properties and two lower bounds. They also proposed a branch-and-bound algorithm and a heuristic algorithm to solve the problem. Ng et al. [26] considered a two-machine flow shop scheduling problem with proportional linear deterioration. The objective is to find a sequence that minimizes the total completion time of the jobs. For the general case, they derived several dominance properties, some lower bounds, and an initial upper bound, and apply them to speed up the elimination process of a branch-and-bound algorithm developed to solve the problem.

In this paper we consider the two-machine flow shop scheduling problem to minimize the total weighted completion time with simple linear deterioration. This model was proposed by Mosheiov [23] and Wang et al. [25]. It is well known that flow shop scheduling to minimize the total completion time is NP-hard even if there are no deteriorating jobs [27]. Therefore, the problem of two-machine flow shop scheduling to minimize total weighted completion time with simple linear deterioration is NP-hard.

For the literature on flow shop scheduling problems without deteriorating jobs, the reader is referred to papers Ignall and Schrage [28], Ho and Gupta [29], Croce et al. [30], Wang et al. [31], Hoogeveen and Kawaguchi [32], Croce et al. [33], Chung et al. [34], Pinedo [35], Lee and Wu [36], Wang et al. [37], Cheng et al. [38], Wang [39], and Easwaran et al. [40].

The rest of the paper is organized as follows. In the next section, we give the problem description. In Section 3 we consider some polynomially solvable special cases. In Section 4 we propose several elimination rules to enhance the efficiency of the search for the optimal solution. In Section 5 we first develop a heuristic algorithm to find near-optimal solutions, then we establish two lower bounds to improve the speed of branching procedures, and finally we propose a branch-and-bound algorithm to search for the optimal solution. In Section 6 we present computational experiments of the branch-and-bound algorithm and the heuristic algorithm. Conclusions are given in the last section.

2. Problem description

Let $N = \{J_1, J_2, \ldots, J_n\}$ be the set of jobs to be scheduled, and $M = \{M_1, M_2\}$ be the two machines. Each job in the set N is processed first on the first machine and then on the second machine. Jobs can only be processed by one machine at a time, and the machines can only process one job at a time. Jobs are processed without interruption or preemption. Both machines are available at all times. The processing time p_{ij} of job $J_j (j = 1, 2, \ldots, n)$ on machine $M_i (i = 1, 2)$ is given as a simple linear increasing function dependent on its execution start time t:

$$p_{ij}(t) = a_{ij}t, \tag{1}$$

where $a_{ij} \in (0, 1)$ denotes the deterioration rate of job J_j on machine M_i. All the jobs are available for processing at time $t_0 > 0$. For each job J_j, a weight w_j indicating its relative importance is given. The objective is to find a schedule that minimizes the total weighted completion time. We assume unlimited intermediate storage between successive machines for the general flow shop scheduling problem.

Let $C_{ij}(\pi)$ denote the completion time of job J_j on machine M_i under some schedule π, and $C_{i[j]}(\pi)$ denote the completion time of the jth job on machine M_i under schedule π. Thus, the completion time of job J_j is $C_j = C_{2j}$. Using the three-field natation for problem classification, the problem can be represented as $F2|a_{ij}t|\sum w_jC_j$. For ease of exposition, we denote a_{1j} by a_j, and a_{2j} by b_j, $j = 1, 2, \ldots, n$. Since unlimited intermediate storage is assumed, clearly an optimal schedule exists with no idle times between consecutive jobs on machine M_1. Therefore, from Mosheiov [8], the completion time of the jth job on machine M_1 is given by:

$$C_{1[j]} = t_0 \prod_{i=1}^{j} (1 + a_{[i]}), \quad j = 1, 2, \ldots, n. \tag{2}$$

3. Solvable cases

Lemma 1. *There exists an optimal schedule in which the job sequence is identical on both machines.*

Proof. Similar to the proof of Lemma 1 in Mosheiov [23]. □

The conclusion of Lemma 1 is that only permutation schedules need be considered for this problem. In what follows, it is shown that our problem is solvable for some special cases.